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BOOK OF ABSTRACTS
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Meeting on Lorentz-Finsler Geometry and Applications

http://gigda.ugr.es/finslermeeting

LORENTZ-FINSLER RELATIVITY

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Abstract
In General Relativity one uses pseudo-Riemannian manifolds of Lorentzian signature as spacetime models. In an introductory part of my talk, I will motivate the idea of generalising this to (pseudo-)Finsler manifolds of Lorentzian signature with the Ehlers-Pirani-Schild axiomatics. In the main part of my talk I will discuss various methods by which one could distinguish such a Lorentz-Finsler Relativity from standard Relativity by way of observations. Among other things, I will discuss planetary motion and the deflection of light in a Lorentz-Finsler model of our Solar system.

THE ORIGIN OF FINSLER-SPACETIME GEOMETRY IN PHYSICS

Christian Pfeifer
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Abstract
From a modern point of view the geometry of spacetime has mainly two origins in physics: it can be derived from an observers clock, which is linked to an observers worldline and the geometry of spacetime via Einstein's clock postulate, or, it emerges from the point particle limit of a fundamental field theory. This reasoning yields pseudo-Riemannian Lorentzian spacetime geometry in the first case by imposing Lorentz transformations as observer transformations, in the second case it can be derived from Maxwell electrodynamics. Changing the notion of observer symmetries or the underlying matter field theories naturally leads to a Finslerian spacetime geometry. In this talk I present how physical systems, like the description of electrodynamics in media and effective models of the interaction of particles and fields with quantum gravity, lead to a Finslerian spacetime geometry. These physical examples lead us to a definition of Finsler spacetimes, a description of Finsler spacetime observers and to a proposal for Finsler generalisations of the Einstein equations which determine the Finslerian spacetime geometry dynamically.

VARIATIONAL PROBLEMS IN PSEUDO-FINSLER SPACES

Nicoleta Vaicu
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Abstract

(Based on joint work with M. Hohmann and
C. Pfeifer, University of Tartu, Estonia)

In classical field theory, fields are treated as sections of a certain fibered manifold (Y, π, X), called the configuration manifold, Lagrangians are treated as differential forms on some bundle J^rY and variations of the action are expressed in terms of Lie derivatives of the Lagrangian with respect to certain vector fields on J^rY.

But, applying this very elegant mathematical apparatus in order to build a Finsler geometry-based field theory is highly non-trivial. Specific problems include the fact that, generally, in a Finsler spacetime, some typical geometric objects (e.g., the metric tensor or an invariant volume form) cannot be defined along all directions in each tangent space, or the non-compactness of the indicatrices.

In the present talk, we introduce and compare some techniques that allow one to correctly define Finslerian actions and their variations, as well as to characterize the invariance of the Lagrangian with respect to certain Lie group actions.

From Lorentz to Lorentz-Finsler Geometry

Miguel Sánchez
University of Granada

Abstract

We will give a brief summary of the geometric framework in the transition from classical Riemann-Finsler to Lorentz-Finsler geometries. This includes a physical motivation on the geometries of spacetimes, the study of cone structures, Lorentz-Finsler metrics and its compatibility, and the discussion of examples and applications.

Most of the talk will be based on joint work with M.A. Javaloyes, arxiv: 1805.06978.

A FINSLERIAN NOTION OF CAUSAL STRUCTURE

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Abstract

Using the relation between Finslerian and Riemannian geometry, we give a microlocal treatment of causal structures as a generalization of conformal pseudo-Riemannian geometry. We use Cartan's method of equivalence to solve the equivalence problem of causal structures and give a geometric interpretation of their fundamental invariants. We will focus on a special classes of causal structures in dimension four and highlight some of their remarkable features.

TIME-DEPENDENT FINSLER GEOMETRY FOR WILDFIRE SPREAD MODELLING

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Abstract

In this talk we first review Gwynfor D. Richards' equations for the parametric spread of ellipse-borne wildfires, [1]. We generalize these equations to cover any type of (possibly time-dependent) ovaloid-borne wildfires in any dimension - dimensions 2 and 3 being the most relevant and interesting, of course. In this setting a given ovaloid at a given point in space at a given time is to be thought of as the local indicatrix, i.e. the local firelet, that is obtained by short (unit)time spread of a model fire from that point, ignited at that given time, and under the assumption of homogeneous (linearized) measures of fuel, wind, and topography. To be precise, the linearization is performed at the given point and time so that the ensuing firelet indicatrix is, geometrically speaking, molded in the tangent space of the wildfire domain. In this way we obtain an ovaloid in each tangent space at each time, i.e. a time-dependent ovaloid field on the domain of the wildfire.

En passant we will briefly indicate how these ovaloids can be explicitly constructed from observations and experiments that only involve well-controlled

and relatively simple line-ignited fires in the respective tangent spaces. Naturally, each ovaloid will contain the tangent space origin (the point of local ignition) in its interior, and it will usually be a strongly convex set in each tangent space. Under this assumption of strong convexity, the ovaloid field is then precisely a time-dependent indicatrix field of a time-dependent Finsler metric F on the domain $\mathscr{U}$ under consideration.

In this Finsler geometric setting the generalized Richards' equations can now be formulated as a time-dependent eikonal type Finsler-Hamilton-Jacobi equation. We show how to solve these equations – and thence the corresponding wildfire spread problem – using results from the differential geometry of Finsler geodesic sprays and/or the control geometry of Finsler differential inclusions, see e.g. [2].

Both methods are readily available and well defined directly via the Finsler metric F on the wildfire domain $\mathscr{U}$. In particular we will emphasise the corresponding inherited motion of what we call *fire particles* (i.e. the Finsler geodesics issuing from a given ignition set) as a natural way to understand both the spread of the frontals of the fire as well as the formation of their ensuing singularities.

In differential geometry the latter singularities are known as cut loci. They correspond to places where fire fighters may experience so-called bear hugs, i.e. where the fire particles, and hence the frontal, approaches from more than one direction. Obviously it is of paramount importance to know how, where, and when such cut loci are formed in each given case.

Finally we will address the important problem of including the curvature of the frontal into a modification of Richards' equations, and more generally into the Finsler eikonal equation. As summarized by Sullivan in [3, p. 162, 166] a point-ignited fire will naturally increase its width but at the same time it will also increase its rate of forward spread. This latter behaviour, and not least the more significant and observed 'straightening out' of concave portions of the fire front (just before a bear hug would otherwise tend to take place), are phenomena, that cannot be explained by a first order eikonal equation. We show how the Finsler eikonal equation mentioned above may be modified into a second order equation, which (by construction) will produce the observed initial increase in fire particle speed from point ignitions. Moreover, the modified equation has speedy fire particle solutions that will also typically 'evaporate' the first encountered segments of the cut loci mentioned above and thence contribute to the straightening of the wildfire frontal.

References

[1] Gwynfor D. Richards. Elliptical growth model of forest fire fronts and its numerical solution. *International Journal for Numerical Methods in Engineering*, 30(6):1163–1179, 1990.

[2] Steen Markvorsen. A Finsler geodesic spray paradigm for wildfire spread modelling. *Nonlinear Anal. Real World Appl.*, 28:208–228, 2016.

[3] Andrew L. Sullivan. Inside the Inferno: Fundamental Processes of Wildland Fire Behaviour. Part 2: Heat Transfer and Interactions. *Current Forestry Reports*, 3(2):150–171, 2017.