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Minicourse: Causality and Legendrian linking.**

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Given a globally hyperbolic spacetime (X^{m+1}, g) put \mathcal{N} to be the space of future directed light rays (null geodesics) in X . The sphere of all light rays through $x \in X$ is called the sky S_x of x . Low observed that \mathcal{N} is a contact manifold and S_x is a Legendrian sphere. Low and later Nataro and Tod conjectured that for many spacetimes causal relations between $x, y \in X$ can be formulated in terms of linking of the pair of skies (S_x, S_y) .

Nemirovski and myself showed that when the universal cover of the Cauchy surface of X is an open manifold, one has that two events $x, y \in X$ are causally related if and only if the Legendrian link (S_x, S_y) is nontrivial. This proves the conjectures of Low; Nataro and Tod and follows from the interesting relation between causality and the natural partial order on the space of Legendrian spheres in \mathcal{N} . A useful tool that often determines that the link (S_x, S_y) is (topologically) nontrivial is the affine linking number introduced in the joint works with Rudyak.

Poincaré conjecture proved by Perelman combined with the above results implies that for $3 + 1$ -dimensional spacetimes linking completely determines causality unless the universal cover of the Cauchy surface is S^3 . Linking does not determine causality in refocussing spacetimes and we discuss relation between refocussing and the Y_l^x -manifolds studied by Bérard-Bergery.