Pinching the Fermat Metric in Stationary Spacetimes

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September 6∼9, 2011

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Given a standard stationary spacetime and a fixed foliation by spacelike slices, there is a canonical Finsler metric of Randers type on the spacelike hypersurfaces, which is called the Fermat metric. The spacelike slices are Cauchy hypersurfaces iff the Fermat metric is (forward and backward) complete. If one can find Riemannian metrics *smaller* than the Fermat metric on all the slices, their completeness imply the completeness of the Fermat metric and, hence, global hyperbolicity of the spacetime. Accordingly, given a globally hyperbolic stationary spacetime and apt Cauchy hypersurfaces as spacelike slices, this ensures the completeness of all Riemannian metrics on the slices which are *bigger* than the Fermat metric. *Smaller* and *bigger* is understood here in the sense of a half-order; e.g. for a Riemannian metric g and a Randers metric F on manifold M we have $\sqrt{g} \leq F :\Leftrightarrow \sqrt{g_x(v,v)} \leq F(x,v), \forall x \in M, v \in T_xM$. We derive several *smaller* and *bigger* Riemannian metrics, which are *optimal* in the sense of finding the biggest metric, that is smaller or the smallest metric that is bigger in a specific situation. Therefore, we pinch the Fermat metric by Riemannian metrics. The new metrics arise from conformal transformations of some canonical choices of Riemannian metrics on the slices. This results in several new necessary and sufficient conditions for the slices to be Cauchy hypersurfaces, based on Riemannian completeness and the growth of functions on the hypersurfaces. Especially we are able to give a sufficient condition for global hyperbolicity only based on the growth of a function. Hence, this constitutes a purely analytic criterion for global hyperbolicity. For more details see [1].

References

[1] A. Dirmeier, M. Plaue, and M. Scherfner, Growth Conditions, Riemannian Completeness and Lorentzian Causality, *to appear in J. Geom. Phys.* (2011),