

Families of parallel CMC hypersurfaces in noncompact rank-one symmetric spaces

José Carlos Díaz-Ramos and Miguel Domínguez-Vázquez

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In any Riemannian ambient manifold, a hypersurface is said to be isoparametric if it and its nearby equidistant hypersurfaces have constant mean curvature. The study of these families of parallel CMC hypersurfaces was started by B. Segre and É. Cartan, who classified these objects in Euclidean and real hyperbolic spaces. In contrast to what happens in these two cases, in spheres there exist isoparametric hypersurfaces which are not homogeneous (that is, orbits of an isometric action). This is one of the reasons why the problem in spheres is much more involved and has motivated many interesting ideas in the last decades.

A remarkable feature of isoparametric hypersurfaces in real space forms is that they have constant principal curvatures. Under certain additional hypotheses, this relation still holds for more general ambient spaces, such as nonflat complex space forms. This allows us to get some partial classifications in complex space forms, based on the results achieved in [2].

However, in general, an isoparametric hypersurface does not have constant principal curvatures. Many examples of this behaviour were found by the authors in [1], for the case of complex hyperbolic spaces.

In this communication, we will explain the extension of the construction in [1] to noncompact rank-one symmetric spaces. As a by-product, we will obtain the first known inhomogeneous isoparametric hypersurface with constant principal curvatures in a space different from a sphere.

References

- [1] J. C. Díaz-Ramos, M. Domínguez-Vázquez, Inhomogeneous isoparametric hypersurfaces in complex hyperbolic spaces, *preprint*, 2010, arXiv:1011.5160v1. To appear in *Math. Z.*
- [2] J. C. Díaz-Ramos, M. Domínguez-Vázquez, Non-Hopf real hypersurfaces with constant principal curvatures in complex space forms, *preprint*, 2009, arXiv:0911.3624v1. To appear in *Indiana Univ. Math. J.*