Lorentzian quasi-Einstein manifolds

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A pseudo-Riemannian manifold (M, g) of dimension $n+2$, $n \geq 1$, is *quasi-Einstein* if there exists a smooth function $f : M \to \mathbb{R}$ such that

$$\rho + \mathrm{Hes}_f - \mu df \otimes df = \lambda g, \qquad (1)$$

where ρ and Hes_f are the Ricci tensor and the Hessian of f, for some constants μ, $\lambda \in \mathbb{R}$ [1], [2]. If the function f is constant we get the Einstein equation and if $\mu = 0$ we obtain the gradient Ricci soliton equation [3]. Moreover, the existence of quasi-Einstein metrics is closely related to the existence of warped product Einstein metrics. Indeed, if $M \times_f F$ is Einstein, then $\phi = -(\dim F) \ln f$ is a quasi-Einstein structure: $\rho + \mathrm{Hes}_\phi - \frac{1}{\dim F} d\phi \otimes d\phi = \lambda g$.

The aim of this work is to investigate locally conformally flat quasi-Einstein Lorentzian manifolds, showing that they are locally isometric to a space form, a warped product of Robertson-Walker type or a locally conformally flat *pp*-wave.

References

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[3] M. Brozos-Vázquez, E. García-Río and S. Gavino-Fernández, Locally conformally flat Lorentzian gradient Ricci solitons, arXiv:1106.2924v1 [math.DG].