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Title: Kähler and para-Kähler Weyl manifolds of dimension 4

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Abstract: Let (M, g) be a pseudo-Riemannian manifold. Let $J_\pm$ be an integrable almost (para)-complex structure, i.e. an endomorphism of the tangent bundle so that $J_\pm^2 = \pm \mathrm{id}$ and so that the associated Nijenhuis tensor vanishes. One assumes $J_\pm^* g = \mp g$; in the para-complex setting necessarily g has neutral signature. The triple $(M, g, J_\pm)$ is then said to be a *para/pseudo-Hermitian manifold*.

A torsion free connection ∇ on the tangent bundle of M is said to be a *Weyl* connection and (M, g, ∇) is said to be a *Weyl structure* if there is a 1-form ϕ so that $\nabla g = -2\phi \otimes g$. We examine the interaction of these two structures. One says that $(M, g, J_\pm, \nabla)$ is a para/pseudo-Kähler Weyl manifold if $\nabla J_\pm = 0$ (this condition automatically implies $J_\pm$ is integrable). In dimension $m \geq 6$, the Weyl structure of any para/pseudo-Kähler Weyl manifold is trivial, i.e. there exists a conformally equivalent metric $\tilde{g}$ so that $(M, \tilde{g}, J_\pm)$ is Kähler and so that ∇ is the Levi-Civita connection determined by $\tilde{g}$. The 4-dimensional setting is very different and forms the focus here: one has that every pseudo-Hermitian manifold of dimension 4 admits a unique Kähler Weyl structure and, similarly, that every para-Hermitian manifold of dimension 4 admits a unique para-Kähler Weyl structure. These results rely upon a curvature decomposition which extends previous results of Higa to the complex setting and also upon an extension of the classical Gray-Hervella decomposition to the indefinite setting.