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Plenary Talk: Polar actions on symmetric spaces of noncompact type

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An isometric action on a Riemannian manifold is said to be *polar* if there exists a submanifold that meets all the orbits of the action orthogonally; such a submanifold is called a *section*. A section is known to be totally geodesic, and if it is flat, the action is said to be *hyperpolar* [4].

Polar actions on Euclidean spaces have been classified by Dadok [5]. The classification of hyperpolar actions on symmetric spaces of compact type follows from the work by Podestà and Thorbergsson in rank one [10], and by Kollross in higher rank [7]. The classification problem for polar actions on symmetric spaces of compact type is still an open problem (see [8]), but it is interesting to emphasize that no examples of polar, non-hyperpolar actions on symmetric spaces of compact type and rank higher that 2 are known.

Polar and hyperpolar actions on symmetric spaces of noncompact type turn out to be much more involved. In some case one can use duality between symmetric spaces of compact and noncompact type to derive classification results [6], [9], but this strategy does not work in general. In fact, there are examples of polar actions in symmetric spaces of noncompact type that have no counterpart in compact type [3]. Some of these examples are polar but not hyperpolar.

Some partial classifications of polar and hyperpolar actions can be obtained for symmetric spaces of noncompact type. The aim of this talk is to present the latest developments in this area [1], [2], [3].

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