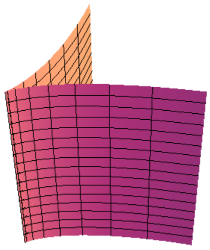
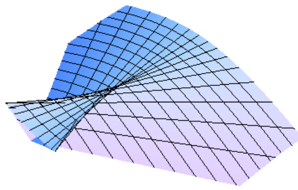
Singularities of the asymptotic completion of developable Möbius strips

Kosuke Naokawa (Tokyo Institute of Technology)

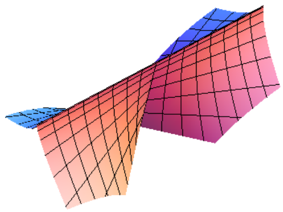
- developable surface = ruled surface & $K = 0$.
- Generic singular points on developable surfaces



cuspidal edge

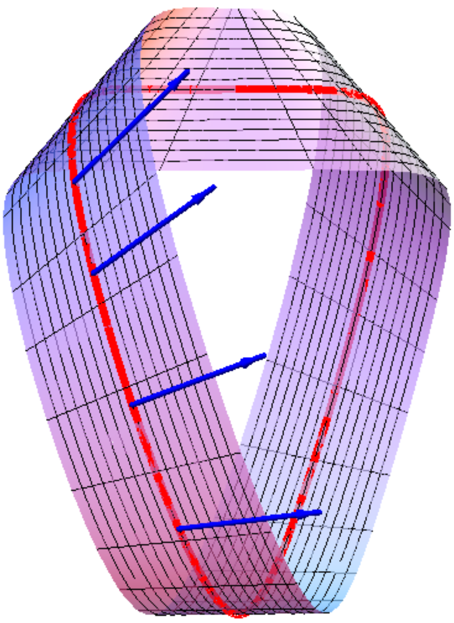


swallowtail



cuspidal cross cap

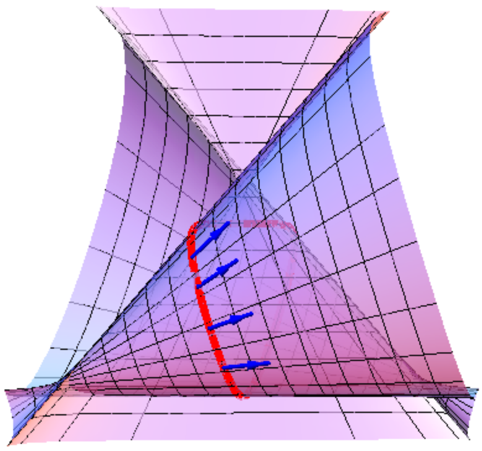
- $F(s,u) = \gamma(s) + u\xi(s)$; a developable Möbius strip



$F(s,u) = \gamma(s) + u\xi(s)$ $(|u| < \epsilon)$

asymptotic completion

$\longrightarrow$



$\tilde{F}(s,u) = \gamma(s) + u\xi(s)$ $(u \in \mathbb{R})$

F;*rectifying Möbius strip*

$:\Longleftrightarrow$ γ; geodesic $(\gamma''(s) \cdot \xi(s) = 0)$

Proposition. The asymptotic completion of a developable Möbius strip has at least one singularity other than cuspidal edges.

Theorem. The asymptotic completion of a rectifying Möbius strip has at least three singularities other than cuspidal edges.

(Outline of a proof of Theorem)

$\kappa(s)$, $\tau(s)$; the curvature & torsion functions of γ,

$\sigma(s) := \tau(s)/\kappa(s)$,

$\xi(s) = \sigma(s)\boldsymbol{e}(s) + \boldsymbol{b}(s)$, where $\boldsymbol{e}(s)$ is the unit tangent vector and $\boldsymbol{b}(s)$ is the binormal vector of $\gamma(s)$.

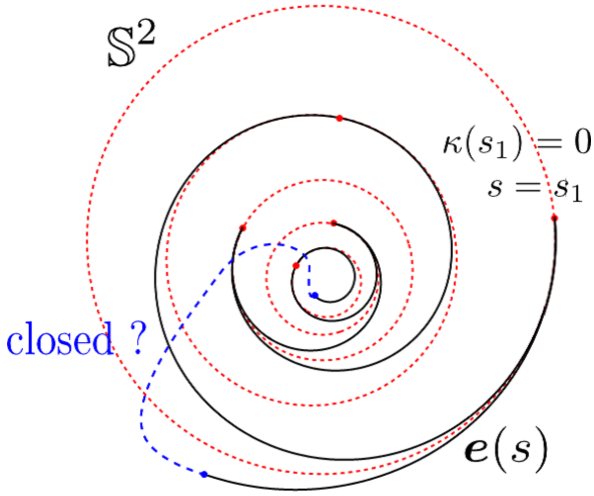
- $\sigma(s)$ can be smoothly across the zeros of κ.
- $\sigma(s)$ = the geodesic curvature function of $\boldsymbol{e}(s)$: $\mathbb{R} \longrightarrow \mathbb{S}^2$ as a spherical curve.
- (s_0, u_0); a singular point of $\tilde{F}$,

$\sigma''(s_0) = 0, \sigma'(s_0) \neq 0 \Rightarrow (s_0, u_0)$; non-cuspidal edge

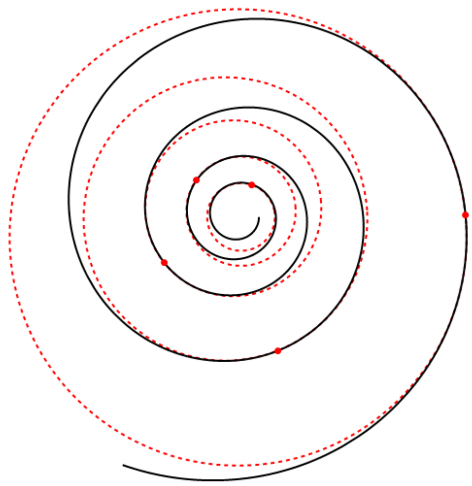
- $\#\{$non-cuspidal edge singularities on $\tilde{F}\}$

$\geq \#\{\sigma''(s) = 0, \sigma'(s) \neq 0\} \geq \#\{\sigma'(s) = 0\}$.

- Assuming $\#\{\sigma'(s) = 0\} = 1 \Rightarrow$ contradiction



The image of $\boldsymbol{e}(s)$



A spiral