Pseudo-umbilical and related surfaces in spacetime

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September 6∼9, 2011

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Consider a spacelike surface S imbedded in a 4-dimensional Lorentzian manifold (V_4, g). S will be said to be umbilical along a direction $\vec{N}$ normal to S if the second fundamental form along that direction is proportional to the first fundamental form of S. In particular, S is pseudo-umbilical if it is umbilical along the mean curvature vector field, and (totally) umbilical if it is umbilical along all possible normal directions.

I will prove that the necessary and sufficient condition for S to be umbilical along a normal direction is that two independent Weingarten operators (and, a fortiori, all of them) commute, or equivalently that the shape tensor be diagonalizable on S. The umbilic direction is then uniquely determined.

This can be seen to be equivalent to a condition relating the normal curvature and the appropriate part of Riemann tensor of the spacetime. In particular, for conformally flat spacetimes (including Lorentz space forms) the neccessary and sufficient condition is that the normal curvature vanishes.

Some furhter consequences will also be analyzed.