Clifford Cohomology of hermitian manifolds

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September 6∼9, 2011

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One of the fundamental objects in the study of a smooth manifold M is its bundle of exterior differential forms, Λ^*M. This is a bundle of algebras over M generated at each point by the cotangent space and in which there is defined a natural first order operator, the exterior differential d. The corresponding fundamental object in the study of a riemannian manifold is its Clifford bundle $Cl(M)$. This is again a bundle of algebras generated by the tangent space at each point, equipped with its inner product, and in which there is another intrinsic first order operator, the Dirac operator D.

In ([1]) Michelsohn uses the Clifford multiplication to elaborate a detailed analysis of Kähler manifolds. For this she considers the bundle $Cl_{\mathbb{C}}(M) = Cl(M) \oplus \mathbb{C}$ and a triple of parallel operators $\mathfrak{L}$, $\overline{\mathfrak{L}}$ and $\mathfrak{H}$ defined on it and which carry an intrinsic $\mathfrak{sl}(2)$-structure of $Cl_{\mathbb{C}}(M)$. This, together with J, yields a decomposition

$$Cl_{\mathbb{C}}(M) \equiv \oplus_{|p+q|\leq n} Cl^{p,q}(M).$$

Taking the hermitian analogues of the Dirac operator D, she obtains operators $\mathfrak{D}$ y $\overline{\mathfrak{D}}$ such that $\mathfrak{D}^2 = \overline{\mathfrak{D}}^2 = 0$, $\mathfrak{D} + \overline{\mathfrak{D}} = 1/2D$ and $\overline{\mathfrak{D}}$ is the formal adjoint of $\mathfrak{D}$. The elements in $Cl^{p,q}$, considered as forms, are generally of mixed degrees and under this assumption the operator $\mathfrak{D}$ corresponds simply to $\overline{\partial} + \partial^*$.

In ([3]) the authors define a formally holomorphic connection over those hermitian manifolds which satisfy the third curvature condition. The expresion for this connection is

$$\nabla_X = \nabla_X^{L.C.} - \frac{1}{2} J(\nabla_X^{L.C.} J)$$

where $\nabla^{L.C.}$ represents the Levi-Civita connection and $X \in T_{\mathbb{C}}M$.

In this contribution we use the algebraic theory of the Clifford algebra $Cl_{\mathbb{C}}(M)$ developed by Michelsohn and this formally holomorphic connection to obtain similar operators to $\mathfrak{D}$ and $\overline{\mathfrak{D}}$, $\mathfrak{D}^{\nabla}$ and $\overline{\mathfrak{D}}^{\nabla}$ respectively, on certain hermitian non Kähler manifolds, and which satisfy similar properties as, for example, $(\mathfrak{D}^{\nabla})^2 = (\overline{\mathfrak{D}}^{\nabla})^2 = 0$ and $\overline{\mathfrak{D}}^{\nabla}$ is the formal adjoint of $\mathfrak{D}^{\nabla}$.

Referencias

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