Closed Geodesics in Lorentzian Surfaces

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G. Galloway proved in [1] that every closed Lorentzian surface contains at least one closed timelike or null geodesic. From the perspective of dynamical systems this result is not optimal. This is because the null geodesics need not be be closed geodesics in the usual sense, i.e. the tangent curves are not closed orbits of the geodesic flow. This is due to the fact that $\dot\gamma(T) \sim \dot\gamma(0)$ does not imply $\dot\gamma(T) = \pm\dot\gamma(0)$ if γ is a lightlike geodesic. This leaves the possibility of closed Lorentzian surfaces without closed geodesics, at least from a dynamical point of view.

My communication will give an overview of the proofs of the following results:

Theorem ([2])**.** *Every closed (i.e. compact and with empty boundary) Lorentzian surface contains two closed geodesics, one of which is definite, i.e. time- or spacelike. This lower bound is optimal.*

Theorem ([2])**.** *The geodesic flow of a closed Lorentzian surface has at least two closed orbits. This lower bound is optimal.*

Corollary ([2])**.** *Every time and space orientable closed Lorentzian surface contains four closed geodesics, two of which must be definite, i.e. every spacetime structure on the* 2*-torus contains at least four closed geodesics.*

Examples attaining the lower bounds will be given. I will conclude by discussing the relationship between the least number of closed geodesics in a closed Lorentzian surface in comparison to the connected component in the space of all Lorentzian metrics on that surface.

References

[1] G. Galloway, Compact Lorentzian Manifolds without Closed Nonspacelike Geodesics, *Proc. Amer. Math. Soc.*, **98**, (1986), 119–123.

[2] S. Suhr, Closed Geodesics in Lorentzian Surfaces, *preprint*, 2010, `arXiv:1011.4878`.