Conformally flat homogeneous Lorentzian manifolds

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This is a joint work with Kyoko Honda (Ochanomizu University).

We consider the problem to classify conformally flat homogeneous semi-Riemannian manifolds.Conformally flat homogeneous Riemannian manifolds were classified by H. Takagi [1].They are all symmetric spaces. While three-dimensional conformally flat homogeneous Lorentzian manifolds were classified by us [2] and the examples which are not symmetric spaces were found. In this talk, we report the result on the classification of higher dimensional conformally flat homogeneous Lorentzian manifolds.

Let M^n be an $n(\geq 4)$ dimensional conformally flat homogeneous Lorentzian manifold. Our classification depends on the form of the modified Ricci operators $A = \frac{1}{n-2}\left(Q - \frac{S}{2(n-1)} Id\right)$, where Q is the Ricci operator and S is the scalar curvature of M, respectively.

Theorem Let M^n be an $n(\geq 4)$ dimensional conformally flat homogeneous Lorentzian manifold. Then the linear operator A has exactly one of the following forms:

Case 1 $\lambda I_r - \lambda I_{n-r}$
Case 2 $\lambda(e_1^* \otimes e_2 - e_2^* \otimes e_1 + \sum_{i=3}^{m+2} e_i^* \otimes e_i - \sum_{i=m+3}^{2m+2} e_i^* \otimes e_i)$. $(\lambda > 0)$
Case 3 $\lambda I_n + \varepsilon e_2^* \otimes e_1$ $(\lambda \leq 0,\ \varepsilon = \pm 1)$
Case 4 $\lambda I_r - \lambda I_{n-r} + e_1^* \otimes e_3 + e_3^* \otimes e_2$ $(3 \leq r \leq n, \lambda < 0)$

where $I_r = \sum_{i=1}^{r} e_i^* \otimes e_i$ and $I_{n-r} = \sum_{i=r+1}^{n} e_i^* \otimes e_i$. Our expressions are those with respect to an orthonormal basis $\langle e_1, e_1 \rangle = -1,\ \langle e_i, e_j \rangle = \delta_{ij}\ (i, j \geq 2)$ in Case 1 and those with respect to a semi-orthonormal basis $\langle e_1, e_2 \rangle = 1,\ \langle e_i, e_j \rangle = \delta_{ij}\ (i, j \geq 3)$ in Cases 2,3, and 4. We denote by $\{e_1^*, e_2^*, \cdots, e_n^*\}$ the dual basis of $\{e_1, e_2, \cdots, e_n\}$.

We construct examples and classify conformally flat homogeneous Lorentzian manifolds for each case.

References

[1] H.Takagi,*Conformally flat Riemannian manifolds admitting a transitive group of isometries*,Tôhoku Math. J., 27(1975), 103-110.

[2] K.Honda and K.Tsukada,*Three-dimensional conformally flat homogeneous Lorentzian manifolds*,J.of Phys A:Math.Theor.,40(2007),831-851.