The geometry of collapsing isotropic fluids

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Joint work in progress with
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Outline

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The problem

Collapsing isotropic fluid

- spherically symmetric
- isotropic pressures
- dynamically forming a singularity in the future

Main motivation

Causal structure (in)stability of solutions producing naked singularities

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Collapsing isotropic fluid

solutions to Einstein Field Equations (EFE)

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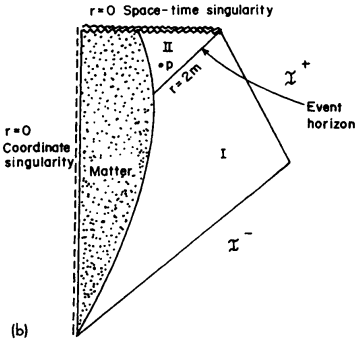

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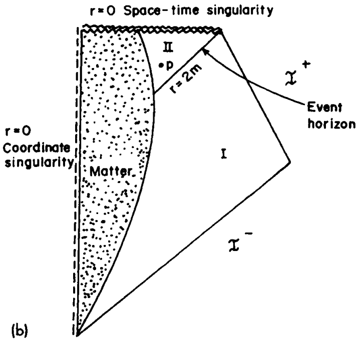
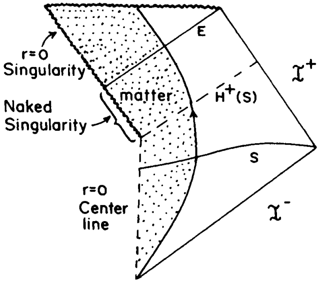

r = 0 Space-time singularity
II
p
r = 2m
Event horizon
r = 0
Coordinate singularity
Matter
I
(b)


E
r = 0
Singularity
matter
$H^+(S)$
Naked Singularity
S
r = 0
Center line

The model

EFE (2^{nd} order PDE of hyperbolic type)

Return

- $g = -e^{2\nu(t,r)}\mathrm{d}t^2 + e^{2\lambda(t,r)}\mathrm{d}r^2 + R(t,r)^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$
- $G^\alpha_\beta = \mathrm{diag}\left(-\rho(t,r), p_r(t,r), p_t(t,r), p_t(t,r)\right)$
- singularity: $R/r = 0$

Isotropic case

$p_r = p_t \; (=: p)$

Coordinate change

- new coordinates: $v := R/r, \quad r$
- $w := \frac{\partial v}{\partial r},\; z := \frac{\partial v}{\partial t}$

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An (The) exact solution: dust cloud (p=0)

"marginally bound" case

- $\nu(v,r) = 0$
- $\lambda(v,r) = \log v + \log\left[1 - \frac{M'(r)r}{3M(r)}\left(\frac{1}{v\sqrt{v}} - 1\right)\right]$
- $\rho(v,r) = \frac{rM'(r)+3M(r)}{2v^2e^\lambda}$

Initial data

$M(r) = M_0 + M_n r^n + o(r^n)$

Remark

$r-$power development of $\lambda(v,r)$ badly behaves as $v \to 0$

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Remark

$r-$power development of $\lambda(v, r)$ badly behaves as $v \to 0$ – unlike e^λ

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Theorem (Joshi and Dwivedi, CMP 1994)

- $M(r) = M_0 + M_n r^n + o(r^n)$
- $n = 1, 2$ or $n = 3$, $|M_3| > \frac{26+15\sqrt{3}}{2} M_0^{5/2} \implies \exists\, \infty$ outgoing null geodesics from the central singularity escaping outside the trapped region

Cosmic censorship

Dust cloud is a toy model providing a counterexample to Penrose conjecture

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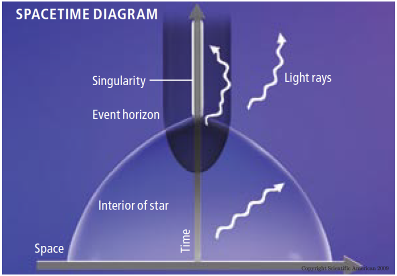

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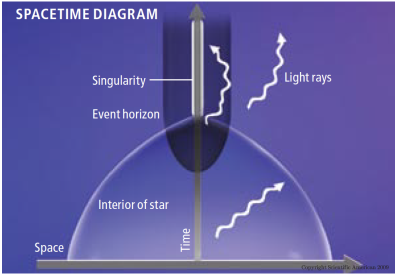

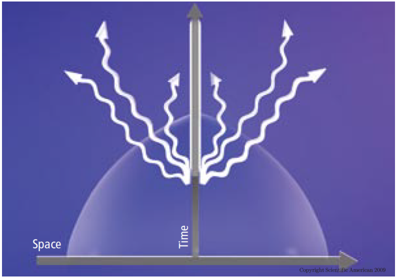

Stepping away...from the dust

Problems

- $p = 0$ very special case (unphysical!)
- naked sing. stability wrt perturbations
- our choice: $p_r = p_t (= p)$
- "genericity" depends on the total space

Genericity, a case study: collapse of a scalar field

- Christodoulou (Ann Math 1994) counterexample to CC
- Christodoulou (Ann Math 1999) instability wrt perturbations
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- Christodoulou (Ann Math 1994) counterexample to CC
- Christodoulou (Ann Math 1999) instability wrt perturbations
- BV perturbations used $\Longrightarrow$ total space is too large!!

Stepping away...from the dust

Isotropic pressures and CC

- almost no collapsing solutions known analitically
  - except some special case, e.g. de Sitter, or under self-similarity assumptions (Ori and Piran PRD 1990, Harada and Maeda PRD 2001, . . .)
- Many numerical studies (but beware code near singularity)
  - Ori and Piran PRL 1987, Harada 1998, Neilsen and Choptuik CQG 2000, Snajdr CQG 2006, . . .
- present literature on analytical CC studies confined almost totally to anisotropic cases
  - e.g. RG, Giannoni, Magli and Piccione, CQG 2003 and CMP 2003
- isotropic pressure $\Rightarrow$ one more constraint

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2^{nd} order PDE in the unknowns M, ρ, Y, F, B of (v, r)

- $3M + M_r r + wrM_v - \rho v^2(wr + v) = 0$
- $(k+1)\rho(wr+v)Y_v + Y[(k\rho)_r + w(k\rho)_v]r = 0$
- $r(F_r + wF_v)Y - (wr+v)Y_v F = 0$
- $z_r + wz_v - zw_v = 0$

- $k = -M_v \rho^{-1} v^{-2}$
- $z = -F\left(\frac{M}{v} + \frac{Y^2 - 1}{r^2}\right)^{1/2}$
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Regularity assumptions

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Return

Relation with the old system

g

- $F = e^{\nu}$, $B = e^{\lambda}$
- $Y = \frac{\partial R}{\partial r} e^{-\lambda}$
- $\frac{1}{2}Mr^3$ Misner-Sharp mass

A first insight

Assumptions and strategy

- assume r- developability, as for dust (reasonable at least for initial data)
- EFE above give algebraic relations for the coefficients in terms of some *free* functions
- fixing gauges and choosing an equation of state (i.e. $p = p(\rho)$) $\Rightarrow$ free functions choice
- info on lower order coefficients allow to study causal structure near $r = 0$ (i.e. near the central singularity)

Crucial issue

Singularity theorem for isotropic fluid from regular initial data is needed

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Crucial issue

Singularity theorem for isotropic fluid from regular initial data is needed ($\Rightarrow$ PDE study) EFE

Counterexamples to almost anything

Barotropic perfect fluid with linear eos

- $p = k\rho,\ k \in [-1, 1] \setminus \{0\}$
- $M(r,v) = v^{-3k}\left(m_0 + \frac{3}{10}(1+k)\left(2v^2 + 5b_2\right)m_0 r^2 + o(r^3)\right)$
- $\rho(r,v) = \frac{3}{v^{3(1+k)}}\left(m_0 + \frac{(1+k)\left(2v^2(-2+3k)+15kb_2\right)m_0 r^2}{10k} + o(r^3)\right)$
- $k < -1/3 \Rightarrow$ horizon does not form near the center
- $k > -1/3 \Rightarrow$ horizon forms and fully covers the singularity up to the centre $\Rightarrow$ BH

Remark

Not defined for $k = 0 \Rightarrow$ linear eos not a proper dust limit

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Nonlinear equation of state

- $p = \rho^{1+\alpha}$, $\alpha < 0$
- $\alpha = -1 \Rightarrow$ BH
- $\alpha = -2 \Rightarrow$ central NS

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Central NS appears also when $p = -1/\rho^2$ (Chaplygin gas)

Conjecture

NS stable up to perturbations of dust EOS such that $\lim_{\rho \to +\infty} p = 0$ only

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- NS survives only when the fluid is asymptotically dust-like
- anisotropic fluid with diverging pressures already known $\Rightarrow$ crucial role of isotropy
- singularity theorems needed (some models excluded?)
- towards a revised version of CC conjecture

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