VI International Meeting on Lorentzian Geometry

Facultad de Ciencias
Universidad de Granada

Granada, 6–9 September, 2011
The Organizing Committee of the *VI International Meeting on Lorentzian Geometry, Granada 2011 (gelogra’11)*, http://gigda.ugr.es/gelogra/, would like to welcome all participants of the meeting. It will be held on September 6–9, 2011 at the Science Faculty of the University of Granada, in the beautiful and monumental city of Granada, located in the south of Spain.

Ten years ago and in a friendly atmosphere, several Spanish research groups interested on the area of Lorentzian Geometry, its applications and related mathematical topics, initiated the biennial Meetings on Lorentzian Geometry and its Applications in Benalmádena-2001 (Málaga). Since that time, this series of meetings has been consolidated, has become international, and fortunately, now we are in the sixth edition of the event.

Lorentzian Geometry was born as the mathematical tool used in General Relativity. Nowadays, it has own identity and constitutes a very active branch of Differential Geometry where many mathematical techniques are involved (Geometric Analysis, Functional Analysis, Partial Differential Equations, Lie groups and Lie algebras, . . .).

In this meeting, topics on pure and applied Lorentzian geometry such as geodesics, submanifolds, causality, black holes, Einstein equations, geometry of spacetimes or AdS-CFT correspondence, will be covered. The Scientific Committee has designed an intensive and interesting program of invited lectures and mini-courses, as well as a selection of short talks. In addition, a wide number of posters will complete the scientific program.

There will also be some cultural and social activities, including a reception at the Hospital Real, which is the main administrative building of the University of Granada, and a enjoyable guided visit to the Alhambra Monument, for those interested participants.
Finally, we would like to wish everybody who is attending GELOGRA’11 a fruitful stay, both geometrically and personally. This has been our main aim in the organization of this meeting.

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Francisco J. Palomo  
Alfonso Romero  
Miguel Sánchez  
Francisco Torralbo  

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Research Group *Differential Geometry and its applications* FQM-324
http://gigda.ugr.es/gigda/

Research project *Lorentz Geometry and Gravitation* P09-FQM-4496
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Research project *Semi-Riemannian Geometry and Variational Problems in Mathematical Physics* MTM2010-18099
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Calabi-Bernstein results and parabolicity of maximal surfaces in Lorentzian product spaces

Luis J. Alías (ljalias@um.es)
Universidad de Murcia

A maximal surface in a 3-dimensional Lorentzian manifold is a spacelike surface with zero mean curvature. Here by *spacelike* we mean that the induced metric from the ambient Lorentzian metric is a Riemannian metric on the surface. The terminology *maximal* comes from the fact that these surfaces locally maximize area among all nearby surfaces having the same boundary. Besides their mathematical interest, maximal surfaces and, more generally, spacelike surfaces with constant mean curvature are also important in General Relativity.

One of the most important global results about maximal surfaces is the Calabi-Bernstein theorem for maximal surfaces in the 3-dimensional Lorentz-Minkowski space $\mathbb{R}^3_1$, which, in parametric version, states that the only complete maximal surfaces in $\mathbb{R}^3_1$ are the spacelike planes. This result can be seen also in a non-parametric form, establishing that the only entire maximal graphs in $\mathbb{R}^3_1$ are the spacelike planes; that is, the only entire solutions to the maximal surface equation

$$\text{Div} \left( \frac{Du}{\sqrt{1 - |Du|^2}} \right) = 0, \quad |Du|^2 < 1$$

on the Euclidean plane $\mathbb{R}^2$ are affine functions.

In this lecture, we present new Calabi-Bernstein results for maximal surfaces immersed into a Lorentzian product space of the form $M^2 \times \mathbb{R}_1$, where $M^2$ is a connected Riemannian surface and $M^2 \times \mathbb{R}_1$ is endowed with the Lorentzian metric $\langle \cdot , \cdot \rangle = \langle \cdot , \cdot \rangle_M - dt^2$. In particular, when $M$ is a (necessarily complete) Riemannian surface with non-negative Gaussian curvature $K_M$, we prove that any complete maximal surface in $M^2 \times \mathbb{R}_1$ must be totally geodesic. Besides, if $M$ is non-flat we conclude that it must be a slice $M \times \{t_0\}, t_0 \in \mathbb{R}$ (here by *complete* it is meant, as usual, that the induced Riemannian metric on the maximal surface from the ambient Lorentzian metric is complete). We prove that the same happens if the maximal surface is complete with respect to the metric induced from the Riemannian product $M^2 \times \mathbb{R}$. This allows us to give also a non-parametric version of the Calabi-Bernstein theorem for
entire maximal graphs in $M^2 \times \mathbb{R}_1$, under the same assumptions on $K_M$. Moreover, we also construct counterexamples which show that our Calabi-Bernstein results are no longer true without the hypothesis $K_M \geq 0$.

On the other hand, we introduce a local approach to our Calabi-Bernstein results, which is based on some parabolicity criteria for maximal surfaces with non-empty smooth boundary in $M^2 \times \mathbb{R}_1$. In particular, we derive that every maximal graph over a starlike domain $\Omega \subseteq M$ is parabolic. This allows us to give an alternative proof of the non-parametric version of the Calabi-Bernstein result for entire maximal graphs in $M^2 \times \mathbb{R}_1$. Finally, we also introduce a second local approach to our results, which is given by means of a local integral inequality for the squared norm of the second fundamental form of the surface.

The results in this lecture are part of our recent research work developed jointly with Alma L. Albujer, and it can be found in the following references:

**References**


Stationary-to-Randers correspondence and convexity

Erasmo Caponio (caponio@poliba.it)
Politecnico di Bari

Stationary-to-Randers correspondence (SRC) is a bijective map between Finsler metrics of Randers type on a smooth manifold $S$ and conformal standard stationary Lorentzian metrics on $S \times \mathbb{R}$. In this talk we focus on some convexity properties in a Randers space (as infinitesimal convexity of a hypersurface, convexity of a large sphere in an asymptotically flat manifold) and on their counterpart by SRC. The talk is based on results contained in the following papers:

References


and on a work in collaboration with A.V. Germinario and M. Sanchez.
Polar actions on symmetric spaces of noncompact type

José Carlos Díaz-Ramos (josecarlos.diaz@usc.es)
Universidad de Santiago de Compostela

An isometric action on a Riemannian manifold is said to be polar if there exists a submanifold that meets all the orbits of the action orthogonally; such a submanifold is called a section. A section is known to be totally geodesic, and if it is flat, the action is said to be hyperpolar [4].

Polar actions on Euclidean spaces have been classified by Dadok [5]. The classification of hyperpolar actions on symmetric spaces of compact type follows from the work by Podestà and Thorbergsson in rank one [10], and by Kollross in higher rank [7]. The classification problem for polar actions on symmetric spaces of compact type is still an open problem (see [8]), but it is interesting to emphasize that no examples of polar, non-hyperpolar actions on symmetric spaces of compact type and rank higher that 2 are known.

Polar and hyperpolar actions on symmetric spaces of noncompact type turn out to be much more involved. In some case one can use duality between symmetric spaces of compact and noncompact type to derive classification results [6], [9], but this strategy does not work in general. In fact, there are examples of polar actions in symmetric spaces of noncompact type that have no counterpart in compact type [3]. Some of these examples are polar but not hyperpolar.

Some partial classifications of polar and hyperpolar actions can be obtained for symmetric spaces of noncompact type. The aim of this talk is to present the latest developments in this area [1], [2], [3].

References


The c-boundary of spacetimes and its related boundaries in Differential Geometry.

Jónatan Herrera (jherrera@agt.cie.uma.es)
Universidad de Málaga

In Lorentzian Geometry, the problem of attaching an ideal boundary to any spacetime has been one of the most controversial topics along the last decades. Recently, the notion of causal boundary has been consistently redefined and supported by an exhaustive analysis. Moreover, the computation of this causal boundary on any standard stationary spacetime suggests a new completion in Riemannian and Finslerian geometries. This completion, called Busemann completion has its own interest and can been compared with more classical completions, such as the Cauchy one for a metric space and the Gromov one for a length space.

The aim of this lecture is threefold: (1) to introduce the new notions of c-boundary and c-completion for strongly causal spacetimes, (2) to introduce the Busemann completion and to compare it with the (extension to arbitrary Finsler manifolds of) Cauchy and Gromov’s one, and (3) to compute systematically the c-boundary of any (standard) static and stationary spacetime by using the Busemann boundary.
### Indefinite extrinsic symmetric spaces

**Ines Kath** ([ines.kath@uni.greifswald.de](mailto:ines.kath@uni.greifswald.de))
Universität Greifswald

We will study symmetric submanifolds of pseudo-Euclidean spaces. A non-degenerate submanifold of a pseudo-Euclidean space is called symmetric submanifold or extrinsic symmetric space if it is invariant under the reflection at each of its affine normal spaces. In particular, each extrinsic symmetric space is an ordinary (abstract) symmetric space. Another characterisation can be obtained in terms of the second fundamental form. Extrinsic symmetric spaces are exactly those connected complete submanifolds whose second fundamental form is parallel. While a nice construction found by Ferus provides a classification of all extrinsic symmetric spaces in Euclidean ambient spaces the pseudo-Riemannian situation is much more involved. We will give a description of extrinsic symmetric spaces in pseudo-Euclidean spaces by corresponding infinitesimal objects and discuss the classification problem for these objects.

### Stability of marginally outer trapped surfaces and applications.

**Marc Mars** ([marc@usal.es](mailto:marc@usal.es))
Universidad de Salamanca

Marginally outer trapped surfaces (MOTS) are associated with strong gravitational fields and generalize the notion of minimal surfaces to a spacetime setting. In this talk I will describe the notion of stability of marginally outer trapped surfaces and I will present several consequences of it. In particular, I will discuss the implications of stability on the spacetime evolution of MOTS, the existence of outermost MOTS in a given spacelike hypersurface, the relationship between stable MOTS and symmetries and an application to a recent area-angular momentum inequality for black holes.
How to reconstruct a metric by its unparameterized geodesics

Vladimir S. Matveev (matveev@minet.uni-jena.de)
Friedrich-Schiller-Universität Jena

We discuss whether it is possible to reconstruct a metric by its unparameterized geodesics, and how to do it effectively. We explain why this problem is interesting for general relativity. We show how to understand whether all curves from a sufficiently big family are unparameterized geodesics of a certain affine connection, and how to reconstruct algorithmically a generic 4-dimensional metric by its unparameterized geodesics. The algorithm works most effectively if the metric is Ricci-flat. We also prove that almost every metric does not allow nontrivial geodesic equivalence, and construct all pairs of 4-dimensional geodesically equivalent metrics of Lorenz signature. If the time allows, I will also explain how this theory helped to solve two problems explicitly formulated by Sophus Lie in 1882, and the semi-Riemannian two-dimensional version of the projective Lichnerowicz-Obata conjecture.

The new results of the talk are based on the papers
joint with Bryant, Bolsinov, Kiosak, Manno, Pucacco
New examples of maximal surfaces in Lorentz Minkowski 3-Space.

Masaaki Umehara
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Tokyo Institute of Technology

Kotaro Yamada
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Tokyo Institute of Technology

Maximal surfaces in Lorentz Minkowski 3-space arise as solutions of the variational problem of locally maximizing the area among spacelike surfaces. By definition, they have everywhere vanishing mean curvature. Like the case of minimal surfaces in Euclidean 3-space, maximal surfaces possess a Weierstrass-type representation formula.

The most significant difference between minimal and maximal surfaces is the fact that the only complete spacelike maximal surfaces are planes. However, if we allow some sorts of singular points for maximal surfaces, the situation changes. The authors showed that if admissible singular points are included, then there is an interesting class of objects called "maxfaces", and introduced notions of completeness and weak completeness.

In this talk, we shall survey the theory of maxfaces, and shall introduce several new examples.
Causality and Legendrian linking

Vladimir Chernov (Vladimir.Chernov@dartmouth.edu)
Dartmouth College

Given a globally hyperbolic spacetime \((X^{m+1},g)\) put \(\mathcal{N}\) to be the space of future directed light rays (null geodesics) in \(X\). The sphere of all light rays through \(x \in X\) is called the sky \(S_x\) of \(x\). Low observed that \(\mathcal{N}\) is a contact manifold and \(S_x\) is a Legendrian sphere. Low and later Natario and Tod conjectured that for many spacetimes causal relations between \(x, y \in X\) can be formulated in terms of linking of the pair of skies \((S_x, S_y)\).

Nemirovski and myself showed that when the universal cover of the Cauchy surface of \(X\) is an open manifold, one has that two events \(x, y \in X\) are causally related if and only if the Legendrian link \((S_x, S_y)\) is nontrivial. Low, and later Natario and Tod, conjectured that for many spacetimes, causal relations between \(x, y \in X\) can be formulated in terms of linking of the pair of skies \((S_x, S_y)\).

Poincaré conjecture, proved by Perelman and combined with the above results, implies that for \(3+1\)-dimensional spacetimes, linking completely determines causality unless the universal cover of the Cauchy surface is \(S^3\). Linking does not determine causality in refocussing spacetimes and we discuss relation between refocussing and the \(Y^2\)-manifolds studied by Bérard-Bergery.
On the isometry group of Lorentz manifolds

Paolo Piccione (piccione.p@gmail.com)
Universidade de São Paulo

The course will consist of three lectures. In the first lecture I intend to discuss a proof of the existence of a Lie group structure in the set of isometries, or more generally of conformal diffeomorphisms, of a semi-Riemannian manifold. In the second lecture I will present the main results of a theory developed mostly by Adam, Stuck and Zeghib in the 90’s, that lead to a classification of Lie groups that act locally faithfully and isometrically on compact Lorentzian manifolds. Finally, in the third lecture, I will present some recent result that have been obtained by Zeghib and myself on the isometry group of compact Lorentz manifolds that admit a somewhere timelike Killing vector field.
Uniqueness and non-existence results for spacelike hypersurfaces with constant mean curvature in $\mathbb{H}^n \times \mathbb{R}_1$

Alma L. Albujer (alma.albujer@uco.es)
Universidad de Córdoba

In this talk we present some uniqueness and non-existence results for complete spacelike hypersurfaces with constant mean curvature in the Lorentzian product space $\mathbb{H}^n \times \mathbb{R}_1$. In order to obtain our results we ask the normal hyperbolic angle of the hypersurface to be bounded in an appropriate way. We get our results as a consequence of the well known Omori-Yau generalized maximum principle for complete Riemannian manifolds.

On the other hand it is well known that a spacelike entire graph in $\mathbb{H}^n \times \mathbb{R}_1$ is not necessarily complete. However, we are also able to state our results in terms of entire graphs. That is, we present non-parametric versions of our results.

This talk is part of joint work with Fernanda E. C. Camargo and Henrique F. de Lima.

References


On the space of geodesics of Riemannian and Lorentzian space forms

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The space of geodesics of a given type (timelike or spacelike) $L^\pm(\mathbb{S}^{n+1}_p)$ of a non-flat pseudo-Riemannian space form $\mathbb{S}^{n+1}_p$ of signature $(p, n + 1 - p)$ and dimension $n$ enjoys a natural invariant structure: in the case of periodic geodesics, it is (pseudo)-Kähler, while in the case of unbounded geodesics, it enjoys a para-Kähler structure.

We prove that the underlying metric of $L^\pm(\mathbb{S}^{n+1}_p)$ has constant scalar curvature, and is moreover Einstein if and only if $\mathbb{S}^{n+1}_p$ is Riemannian (thus the sphere or the hyperbolic space) or Lorentzian (thus the Sitter or anti de Sitter spaces), and the geodesics are timelike.

We also study the normal congruence of a hypersurface $S$ of $\mathbb{S}^{n+1}_p$, which happens to be a Lagrangian submanifold $\tilde{S}$ of $L^\pm(\mathbb{S}^{n+1}_p)$, and relate the geometries of $S$ and $\tilde{S}$. In particular $\tilde{S}$ is totally geodesic if and only if $S$ has parallel second fundamental form. In the Riemannian and spacelike Lorentzian cases, we express the minimality of $\tilde{S}$ by a functional relation involving the principal curvatures of $S$. 
Geodesic connectedness on Gödel type spacetimes: a “static” variational set-up

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We show that the problem of the geodesic connectedness of a wide class of Gödel type spacetimes can be reduced to the search of critical points of a functional naturally involved in the study of geodesics in standard static spacetimes. Then, the accurate estimates in [2] allow us to improve substantially the result in [3], by a weakening of the boundedness assumptions about the metric coefficients.

References


Characterization and structure of second-order symmetric Lorentzian manifolds

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Recently (arXiv:1101.5503v2), we have characterized the second-order symmetric Lorentzian manifolds (i.e., those which satisfy $\nabla^2 Rie = 0$). In particular, all the complete simply-connected non-locally symmetric ones, constitute an extension of the Cahen-Wallach family of ($n$-dimensional) plane waves. Here, we will sketch the techniques for both, local and global results.
Holonomy groups were introduced by Élie Cartan [3] in 1926 in order to study the Riemannian symmetric spaces and since then the classification of holonomy groups has remained one of the classical problems in differential geometry.

**Definition.** Let $M$ be a smooth manifold and $\nabla$ an affine connection on $TM$. The holonomy group of $\nabla$ is a subgroup $\text{Hol}(\nabla) \subset \text{GL}(T_xM)$ that consists of the linear operators $A : T_xM \to T_xM$ being “parallel transport transformations” along closed loops $\gamma$ with $\gamma(0) = \gamma(1) = x$.

In the case of Levi-Civita connections on Riemannian manifolds, the classification of holonomy groups is due to M. Berger [1]. In the pseudo-Riemannian case, the description of holonomy groups is a very difficult problem which still remains open and even particular examples are of interest. We refer to [4], [5] for more information on recent development in this field.

Since we deal with Levi-Civita connections only, we consider subgroups of the (pseudo)-orthogonal group $\text{SO}(g)$, where $g$ is a non-degenerate inner product on a finite-dimensional real vector space $V$. The main result of our paper is

**Theorem.** For every $g$-symmetric operator $L : V \to V$, the identity component of its centraliser in $\text{SO}(g)$

$$G_L = \{ X \in \text{SO}(g) \mid XL = LX \}$$

is a holonomy group for some (pseudo)-Riemannian metric.

In the Riemannian case this theorem becomes trivial as $L$ is diagonalisable and $G_L$ is isomorphic to the direct product $\text{SO}(k_1) \times \cdots \times \text{SO}(k_m) \subset \text{SO}(n)$, which is, of course, a holonomy group. In the pseudo-Riemannian case, $L$ may have non-trivial Jordan blocks and the structure of $G_L$ becomes more complicated.

As usual, instead of $G_L$ we consider its Lie algebra $\mathfrak{g}_L$. The first step is to prove that $\mathfrak{g}_L$ is a Berger algebra. To that end, it is sufficient to construct a formal curvature tensor $R : \Lambda^2(V) \to \text{so}(g)$ such that the image of $R$ coincides with $\mathfrak{g}_L$. The next step is to realise $\mathfrak{g}_L$ as a holonomy Lie algebra for a suitable metric.
Our proof is based on unexpected relationship between curvature tensors for some special metrics and the theory of integrable systems on semi-simple Lie algebras (see [2]). The following two “magic formulas” play crucial role in our construction:

$$R(X) = \frac{d}{dt} \bigg|_{t=0} p_{\text{min}}(L + t \cdot X), \quad X \in \text{so}(g) \simeq \Lambda^2 V,$$

\[ (*) \]

$$C = R(\otimes) = \frac{d}{dt} \bigg|_{t=0} p_{\text{min}}(L + t \cdot \otimes),$$

\[ (**) \]

where \( p_{\text{min}}(\lambda) \) is the minimal polynomial of \( L \). The first formula gives a map \( R : \Lambda^2 V \to \text{gl}(V) \). The second one defines a tensor \( C \) of type (2, 2) whose meaning can be understood from the following example: if \( p_{\text{min}}(\lambda) = \lambda^m \), then

$$R(X) = \sum_{k=1}^{m} L^{k-1} X \Lambda^{m-k-1} \quad \text{and} \quad C = R(\otimes) = \sum_{k=1}^{m} L^{k-1} \otimes \Lambda^{m-k-1}.$$

These two algebraic objects possess the following remarkable properties:

**Proposition.** \( R \) satisfies the Bianchi identity and its image is contained in \( \text{gl}_L \). In other words, \( R \) is a formal curvature tensor for the Lie algebra \( \text{gl}_L \).

So far \( L \) and \( g \) were defined on a fixed vector space \( V \) which now will be considered as \( T_0 \mathbb{R}^n \). We now extend them onto a neighborhood of \( 0 \in \mathbb{R}^n \).

**Proposition.** In local coordinates \((x^1, \ldots, x^n)\), we set \( L = L(0) = \text{const} \) and

$$g = g_{ij}(x) = g_{ij}(0) + \frac{1}{2} \sum_{\alpha, \beta} g_{i\alpha}(0) g_{p\beta}(0) C_{j,\alpha}^{\beta} x^p x^q$$

Then \( L \) is \( g \)-symmetric, \( \nabla L = 0 \) and the curvature tensor for \( g \) at the origin \( 0 \in \mathbb{R}^n \) is \( R(X) \) defined by \((*)\).

Using a kind of “block-wise” modification of formulas \((*)\) and \((**)\), it is not hard to construct a formal curvature tensor \( R(X) \) and \( (2, 2) \)-tensor \( C \) for which Propositions 1 and 2 still hold, but \( R \) is, in addition, an “onto” map. This obviously completes the proof.

**References**

Uniqueness of spacelike hypersurfaces of constant mean curvature in cosmological models with certain symmetries

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In general relativity, symmetry is usually based on the assumption of the existence of a one-parameter group of transformations generated by a Killing or, more generally, conformal vector field. Although it is not always assumed the same causal character for the infinitesimal symmetry, the timelike choice is natural, since the integral curves of such a timelike infinitesimal symmetry provide a privileged class of observers in the spacetime. The presence of such a vector field is not enough to prevent the existence of closed causal curves. However, if the timelike conformal vector field is globally the gradient of some smooth function, then the (clearly noncompact) spacetime admits a global time function. Therefore, it is stably causal. Finally, spacetimes with a timelike gradient conformal vector field (GCS spacetimes) have another interesting property, they admit a foliation by constant mean curvature (CMC) spacelike hypersurfaces.

The aim of this talk is to introduce a new technique to study CMC spacelike hypersurfaces in GCS spacetimes and to give new uniqueness results for compact CMC spacelike hypersurfaces in these ambient spacetimes, both in the parametric and nonparametric case. As an application, the leaves of the natural spacelike foliation of such spacetimes are characterized in some relevant cases.

This talk is based on a joint work with Alfonso Romero and Rafael M. Rubio.

REFERENCES

Pinching the Fermat metric in stationary spacetimes

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Given a standard stationary spacetime and a fixed foliation by spacelike slices, there is a canonical Finsler metric of Randers type on the spacelike hypersurfaces, which is called the Fermat metric. The spacelike slices are Cauchy hypersurfaces iff the Fermat metric is (forward and backward) complete. If one can find Riemannian metrics smaller than the Fermat metric on all the slices, their completeness imply the completeness of the Fermat metric and, hence, global hyperbolicity of the spacetime. Accordingly, given a globally hyperbolic stationary spacetime and apt Cauchy hypersurfaces as spacelike slices, this ensures the completeness of all Riemannian metrics on the slices which are bigger than the Fermat metric. Smaller and bigger is understood here in the sense of a half-order; e.g. for a Riemannian metric $g$ and a Randers metric $F$ on manifold $M$ we have $\sqrt{g} \leq F :\Leftrightarrow \sqrt{g_x(v,v)} \leq F(x,v), \forall x \in M, v \in T_xM$.

We derive several smaller and bigger Riemannian metrics, which are optimal in the sense of finding the biggest metric, that is smaller or the smallest metric that is bigger in a specific situation. Therefore, we pinch the Fermat metric by Riemannian metrics. The new metrics arise from conformal transformations of some canonical choices of Riemannian metrics on the slices. This results in several new necessary and sufficient conditions for the slices to be Cauchy hypersurfaces, based on Riemannian completeness and the growth of functions on the hypersurfaces. Especially we are able to give a sufficient condition for global hyperbolicity only based on the growth of a function. Hence, this constitutes a purely analytic criterion for global hyperbolicity. For more details see [1].

References

Families of parallel CMC hypersurfaces in noncompact rank-one symmetric spaces

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In any Riemannian ambient manifold, a hypersurface is said to be isoparametric if it and its nearby equidistant hypersurfaces have constant mean curvature. The study of these families of parallel CMC hypersurfaces was started by B. Segre and É. Cartan, who classified these objects in Euclidean and real hyperbolic spaces. In constrast to what happens in these two cases, in spheres there exist isoparametric hypersurfaces which are not homogeneous (that is, orbits of an isometric action). This is one of the reasons why the problem in spheres is much more involved and has motivated many interesting ideas in the last decades.

A remarkable feature of isoparametric hypersurfaces in real space forms is that they have constant principal curvatures. Under certain additional hypotheses, this relation still holds for more general ambient spaces, such as nonflat complex space forms. This allows us to get some partial classifications in complex space forms, based on the results achieved in [2].

However, in general, an isoparametric hypersurface does not have constant principal curvatures. Many examples of this behaviour were found by the authors in [1], for the case of complex hyperbolic spaces.

In this communication, we will explain the extension of the construction in [1] to noncompact rank-one symmetric spaces. As a by-product, we will obtain the first known inhomogeneous isoparametric hypersurface with constant principal curvatures in a space different from a sphere.

References

Totally geodesic submanifolds in Robertson-Walker spaces with flat fibers

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We determine all totally geodesic submanifolds of Robertson-Walker spaces $-I \times f \mathbb{R}^n$. We write down the differential equations for a totally geodesic submanifold of such a space and, in the nontrivial case $f' \neq 0$, we derive the necessary condition for the function $f$. We solve explicitly the corresponding systems of equations.

References

In this paper, Lorentzian Euler Savary formula (giving the relation between the curvatures of the trajectory curves drawn by the points of the moving plane in the fixed plane) during one parameter Lorentzian planar motion is taken into consideration. By using an original geometrical interpretation of Lorentzian Euler Savary formula, Lorentzian Bobillier formula is established.

However, another presentation is made in this paper without using the Euler Savary formula. Then the Lorentzian Euler Savary formula will appear as a particular case of Bobillier formula and as a result of the direct way chosen, this new Lorentzian formula (Bobillier) could be considered as a fundamental law in a planar Lorentzian motion in place of Euler Savary’s.

References

Lorentzian quasi-Einstein manifolds

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A pseudo-Riemannian manifold \((M, g)\) of dimension \(n + 2, n \geq 1\), is quasi-Einstein if there exists a smooth function \(f : M \to \mathbb{R}\) such that

\[
\rho + \text{Hes}_f - \mu df \otimes df = \lambda g,
\]

where \(\rho\) and \(\text{Hes}_f\) are the Ricci tensor and the Hessian of \(f\), for some constants \(\mu, \lambda \in \mathbb{R}\) [1], [2]. If the function \(f\) is constant we get the Einstein equation and if \(\mu = 0\) we obtain the gradient Ricci soliton equation [3]. Moreover, the existence of quasi-Einstein metrics is closely related to the existence of warped product Einstein metrics. Indeed, if \(M \times_f F\) is Einstein, then \(\phi = -(\dim F) \ln f\) is a quasi-Einstein structure:

\[
\rho + \text{Hes}_\phi - \frac{1}{\dim F} df \otimes df = \lambda g.
\]

The aim of this work is to investigate locally conformally flat quasi-Einstein Lorentzian manifolds, showing that they are locally isometric to a space form, a warped product of Robertson-Walker type or a locally conformally flat \(pp\)-wave.

References


The geometry of collapsing isotropic fluids

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The study of spacetimes modeling a collapsing spherical isotropic fluid has always been a recurrent topic for relativists – its connection with Tolman–Bondi solution, one of the few known-in-details solutions dynamically collapsing to a singularity, makes it one the most intriguing problem in gravitational collapse. Some results are known from numerical relativity but very few is known about the geometry of the solutions – whether a singularity is developed, and if that is the case, what is the causal structure of the solution. I will sketch a recent line of research aiming to shed new light on the above aspects.

Joint research (work in progress) with: Fabio Giannoni (Camerino), Giulio Magli (Milan Politecnico), Daniele Malafarina and Pankaj S Joshi (Tata Institute, Mumbai)

References

Kähler and para-Kähler Weyl manifolds of dimension 4

Let \((M, g)\) be a pseudo-Riemannian manifold. Let \(J_{\pm}\) be an integrable almost (para)-complex structure, i.e. an endomorphism of the tangent bundle so that \(J_{\pm}^2 = \pm \text{id}\) and so that the associated Nijenhuis tensor vanishes. One assumes \(J_{\pm}^* g = \mp g\); in the para-complex setting necessarily \(g\) has neutral signature. The triple \((M, g, J_{\pm})\) is then said to be a para/pseudo-Hermitian manifold.

A torsion free connection \(\nabla\) on the tangent bundle of \(M\) is said to be a Weyl connection and \((M, g, \nabla)\) is said to be a Weyl structure if there is a 1-form \(\phi\) so that \(\nabla g = -2\phi \otimes g\). We examine the interaction of these two structures. One says that \((M, g, J_{\pm}, \nabla)\) is a para/pseudo-Kähler Weyl manifold if \(\nabla J_{\pm} = 0\) (this condition automatically implies \(J_{\pm}\) is integrable). In dimension \(m \geq 6\), the Weyl structure of any para/pseudo-Kähler Weyl manifold is trivial, i.e. there exists a conformally equivalent metric \(\tilde{g}\) so that \((M, \tilde{g}, J_{\pm})\) is Kähler and so that \(\nabla\) is the Levi-Civita connection determined by \(\tilde{g}\). The 4-dimensional setting is very different and forms the focus here: one has that every pseudo-Hermitian manifold of dimension 4 admits a unique Kähler Weyl structure and, similarly, that every para-Hermitian manifold of dimension 4 admits a unique para-Kähler Weyl structure. These results rely upon a curvature decomposition which extends previous results of Higa to the complex setting and also upon an extension of the classical Gray-Hervella decomposition to the indefinite setting.
Extrinsically flat surfaces of space forms and the geometric structure on the space of oriented geodesics

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It is well known that any complete extrinsically flat surface in the 3-sphere $S^3$ must be totally geodesic, where “extrinsically flat” means vanishing of the Gauss-Kronecker curvature. However, if one admits some singularities, there are many non-trivial complete extrinsically flat surfaces in $S^3$. In this talk, we shall introduce a representation formula for extrinsically flat surfaces in $S^3$ in terms of a pair of two curves in the 2-sphere $S^2$.

The main idea for the representation formula is the correspondence between ruled surfaces in $S^3$ and curves in $\mathcal{L}(S^3)$, where $\mathcal{L}(S^3)$ is the space of oriented geodesics in $S^3$. Then we use the metric on $\mathcal{L}(S^3)$ associated to the minitwistor complex structure.

We shall also introduce some related results for anti-de Sitter 3-space $H_3^1$.

References

Spacelike hypersurfaces of constant higher order mean curvature in generalized Robertson-Walker spacetimes

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Spacelike hypersurfaces in spacetimes are objects of increasing interest in recent years, both from a physical and a mathematical point of view. A basic question on this topic is the problem of uniqueness of spacelike hypersurfaces with constant mean curvature in certain spacetimes, and, more generally, that of spacelike hypersurfaces with constant higher order mean curvature.

In a recent paper, Alías and Colares [1] studied in depth the problem of uniqueness for compact spacelike hypersurfaces with constant higher order mean curvature in spatially closed generalized Robertson-Walker spacetimes. Their approach was based on the use of the so called Newton transformations $P_k$ and their associated second order differential operators $L_k$, as well as on the application of some general Minkowski integral formulae for compact hypersurfaces. In this talk, which is based on the work in [3], we go deeper into this study. We consider first the case of compact spacelike hypersurfaces, completing some previous results given in [1] and we next extend these results to the complete noncompact case. Our approach is based on the use of a generalized version of the Omori-Yau maximum principle for trace type differential operators which includes the operators $L_k$ that has been recently introduced by the authors in [2] for the study of hypersurfaces in Riemannian warped products.

This is a joint work with L. J. Alías and M. Rigoli.

References


Area-Angular momentum inequality in stable marginally trapped surfaces

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We discuss the area-angular momentum inequality $A \geq 8\pi |J|$ and show that it holds for axially symmetric closed outermost stably marginally trapped surfaces. Such surfaces are sections of dynamical trapping horizons, namely hypersurfaces that provide quasi-local models of black hole horizons in otherwise generic Lorentzian manifolds whose Einstein tensor satisfies a dominant energy condition. This inequality represents a particular example of the extension to a Lorentzian setting of tools employed in the discussion of minimal surfaces in a Riemannian context.

K-conformal maps of a stationary spacetime and Randers metric

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We study the conformal maps of a stationary spacetime fixing the flow lines of the timelike Killing field $K$, which we call $K$-conformal maps. We will use the relation between a stationary spacetime $(\mathbb{R} \times S, g)$ and a Randers metric in the base $S$ to study these maps. In particular, $K$-conformal maps project to maps in $S$ that preserves the Randers metric up to the differential of a function. We will deduce some results about the genericity of these maps.
Classification of symmetric M-theory backgrounds

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We classify symmetric backgrounds of eleven-dimensional supergravity up to local isometry. In other words, we classify triples \((M,g,F)\), where \((M,g)\) is an eleven-dimensional lorentzian locally symmetric space and \(F\) is an invariant 4-form, satisfying the equations of motion of eleven-dimensional supergravity. The possible \((M,g)\) are given either by (not necessarily nondegenerate) Cahen–Wallach spaces or by products \(AdS_d \times M^{11-d}\) for \(d = 2, ..., 7\) and \(M^{11-d}\) a not necessarily irreducible riemannian symmetric space. For each such geometry we determine the possible \(Fs\). The backgrounds are grouped in families sharing the same \(F\)-moduli space, which we determine completely in most cases.

On the geometric structure of Ori spacetimes

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Starting in 1993 A. Ori published three spacetimes violating the chronology condition, being close to physical reality in some sense. We would like to reveal the differential geometric structure of these models, i.e. CTCs in detail, geodesics and symmetry and we will construct a new model of Ori-type starting from a Kerr metric. The origin of this talk is an article together with J. Dietz and A. Dirmeier.
Consider a spacelike surface $S$ imbedded in a 4-dimensional Lorentzian manifold $(V_4, g)$. $S$ will be said to be umbilical along a direction $\vec{N}$ normal to $S$ if the second fundamental form along that direction is proportional to the first fundamental form of $S$. In particular, $S$ is pseudo-umbilical if it is umbilical along the mean curvature vector field, and (totally) umbilical if it is umbilical along all possible normal directions.

I will prove that the necessary and sufficient condition for $S$ to be umbilical along a normal direction is that two independent Weingarten operators (and, a fortiori, all of them) commute, or equivalently that the shape tensor be diagonalizable on $S$. The umbilic direction is then uniquely determined.

This can be seen to be equivalent to a condition relating the normal curvature and the appropriate part of Riemann tensor of the spacetime. In particular, for conformally flat spacetimes (including Lorentz space forms) the necessary and sufficient condition is that the normal curvature vanishes.

Some further consequences will also be analyzed.

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According to a general principle in comparison theory, the existence of a line -that is, a causal globally maximizing and complete geodesic- is incompatible with standard energy conditions, except in very special cases. Thus, the geometry of a spacetime in which a line can be formed without violating appropriate energy conditions is very special. This is the basis of the so called Splitting Theorems in Lorentzian Geometry. Of special interest are those splitting theorems in which the hypothesis are enough to guarantee constant curvature of spacetime. In this talk we present an overview of the classical results as well as recent developments in the context of asymptotically flat and asymptotically de Sitter spacetimes.
Closed Geodesics in Lorentzian Surfaces

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G. Galloway proved in [1] that every closed Lorentzian surface contains at least one closed timelike or null geodesic. From the perspective of dynamical systems this result is not optimal. This is because the null geodesics need not be closed geodesics in the usual sense, i.e. the tangent curves are not closed orbits of the geodesic flow. This is due to the fact that $\dot{\gamma}(T) \sim \dot{\gamma}(0)$ does not imply $\dot{\gamma}(T) = \pm \dot{\gamma}(0)$ if $\gamma$ is a lightlike geodesic. This leaves the possibility of closed Lorentzian surfaces without closed geodesics, at least from a dynamical point of view.

My communication will give an overview of the proofs of the following results:

**Theorem** ([2]). Every closed (i.e. compact and with empty boundary) Lorentzian surface contains two closed geodesics, one of which is definite, i.e. time- or spacelike. This lower bound is optimal.

**Theorem** ([2]). The geodesic flow of a closed Lorentzian surface has at least two closed orbits. This lower bound is optimal.

**Corollary** ([2]). Every time and space orientable closed Lorentzian surface contains four closed geodesics, two of which must be definite, i.e. every spacetime structure on the 2-torus contains at least four closed geodesics.

Examples attaining the lower bounds will be given. I will conclude by discussing the relationship between the least number of closed geodesics in a closed Lorentzian surface in comparison to the connected component in the space of all Lorentzian metrics on that surface.

**References**


Conformally flat homogeneous Lorentzian manifolds

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This is a joint work with Kyoko Honda (Ochanomizu University).

We consider the problem to classify conformally flat homogeneous semi-Riemannian manifolds. Conformally flat homogeneous Riemannian manifolds were classified by H. Takagi [1]. They are all symmetric spaces. While three-dimensional conformally flat homogeneous Lorentzian manifolds were classified by us [2] and the examples which are not symmetric spaces were found. In this talk, we report the result on the classification of higher dimensional conformally flat homogeneous Lorentzian manifolds.

Let $M^n$ be an $n$($\geq 4$) dimensional conformally flat homogeneous Lorentzian manifold. Our classification depends on the form of the modified Ricci operators $A = \frac{1}{n-2} (Q - \frac{S}{2(n-1)} Id)$, where $Q$ is the Ricci operator and $S$ is the scalar curvature of $M$, respectively.

**Theorem.** Let $M^n$ be an $n$($\geq 4$) dimensional conformally flat homogeneous Lorentzian manifold. Then the linear operator $A$ has exactly one of the following forms:

**Case 1** $\lambda I_r - \lambda I_{n-r}$
**Case 2** $\lambda (e_1^* \otimes e_2 - e_2^* \otimes e_1 + \sum_{i=3}^{m+2} e_i^* \otimes e_i - \sum_{i=m+3}^{2m+2} e_i^* \otimes e_i).$ ($\lambda > 0$)
**Case 3** $\lambda I_r + \epsilon e_2^* \otimes e_1$ ($\lambda \leq 0, \epsilon = \pm 1$)
**Case 4** $\lambda I_r - \lambda I_{n-r} + e_1^* \otimes e_3 + e_3^* \otimes e_2$ ($3 \leq r \leq n, \lambda < 0$)

where $I_r = \sum_{i=1}^{r} e_i^* \otimes e_i$ and $I_{n-r} = \sum_{i=r+1}^{n} e_i^* \otimes e_i$. Our expressions are those with respect to an orthonormal basis $\langle e_1, e_1 \rangle = -1, \langle e_i, e_j \rangle = \delta_{ij}$ ($i, j \geq 2$) in Case 1 and those with respect to a semi-orthonormal basis $\langle e_1, e_2 \rangle = 1, \langle e_i, e_j \rangle = \delta_{ij}$ ($i, j \geq 3$) in Cases 2, 3, and 4. We denote by $\{e_1^*, e_2^*, \cdots, e_n^*\}$ the dual basis of $\{e_1, e_2, \cdots, e_n\}$.

We construct examples and classify conformally flat homogeneous Lorentzian manifolds for each case.

**References**

Slant geometry of spacelike hypersurfaces in the lightcone with respect to the $\phi$-hyperbolic duals

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One-parameter families of Legendrian dualities which are the extensions of four dualities obtained in [5] for pseudo-spheres in Lorentz-Minkowski Space were given in [6]. In this talk, as an application of such extensions, a new extrinsic differential geometry on spacelike hypersurfaces in the lightcone is constructed with respect to the $\phi$-Hyperbolic Duals [7].

References

Beltrami-Meusnier Formulas of Semi Ruled Surface

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In this paper, we study the sectional curvatures of the generalized semi ruled surfaces in semi Euclidean space $E_{n+1}^n$. The first fundamental form and the metric coefficients of the generalized semi ruled surfaces are calculated and in these regards, Riemannian-Christoffel curvatures are obtained by the help of Christoffel Symbols. So, the curvatures of arbitrary nondejenere tangential sections of the generalized semi ruled surface is investigated. In addition to this, the relations between the sectional curvatures of semi ruled surfaces are obtained. These are called semi-Euclidean Beltrami-Meusnier formulas.

References


About a new type of null Osserman condition on Lorentz $S$-manifolds

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The Osserman conjecture, introduced in [3] for Riemannian manifolds, relates the properties of the Riemannian curvature to the spectral behaviour of the Jacobi operator. The Osserman problem has been partially solved in the Riemannian case and, while it still remains open in the semi-Riemannian context, a complete solution has been reached in the Lorentzian case. As a consequence, in [1] the authors defined a different type of Osserman conditions: the null Osserman conditions with respect to a unit timelike vector tangent to a Lorentz manifold (see also [2]).

It is natural to study the null Osserman conditions in a Lorentz almost contact manifold and, more generally, in a Lorentz $g.f.f$-manifold, where one finds that none of the above types of Osserman conditions can be satisfied.

This motivated us to introduce another kind of null Osserman condition, adapted to the case of Lorentz $g.f.f$-structures, that we called $\varphi$-null Ossermann condition, which the present talk deals with. We study the links between the $\varphi$-null Osserman condition and the behaviour of the $\varphi$-sectional curvature, with a particular attention towards the relationships between the constancy of the $\varphi$-sectional curvature and the $\varphi$-null Osserman condition in a Lorentz $S$-manifold. We state an algebraic characterization of $\varphi$-null Osserman Lorentz $S$-manifolds with two characteristic vector fields, finally giving some examples of such manifolds.

References

We present some results obtained while studying certain properties of the Jacobi curvature operator ([3]), which are related with a new condition of Osserman-type ($\varphi$-null Osserman condition), recently introduced in [3] in a context involving Lorentzian $g.f.f$-manifolds, together with semi-Riemannian submersions.

Namely, on a Lorentz $S$-manifold $(M, \varphi, \xi, \eta^\alpha, g)$, we first find a relationship between the $\varphi$-null Osserman condition and the classical Osserman condition ([3]), with respect to a unit tangent vector in $\text{Im}(\varphi)$ at a point $p \in M$. Then we use this result to examine some properties of projectability involving the $\varphi$-null Osserman condition in (toroidal) principal bundles with $\varphi$-null Osserman Lorentzian $S$-manifolds as total space, and base space which can be either a Lorentzian Sasakian manifold or a Kählerian manifold (see also [1]). Some examples are given at the end.

References

The existence of homogeneous geodesics in homogeneous pseudo-Riemannian and affine manifolds

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It is well known that any homogeneous Riemannian manifold admits at least one homogeneous geodesic through each point. For the pseudo-Riemannian case, even if we assume reductivity, this existence problem was open. We shall use a purely affine method to the existence problem. As the main result we prove that any homogeneous affine manifold admits at least one homogeneous geodesic through each point. As an immediate corollary, we have the same result for the subclass of all homogeneous pseudo-Riemannian manifolds.

References

Extrinsically flat surfaces of space forms and the geometric structure on the space of oriented geodesics

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It is well known that any complete extrinsically flat surface in the 3-sphere $S^3$ must be totally geodesic, where “extrinsically flat” means vanishing of the Gauss-Kronecker curvature. However, if one admits some singularities, there are many non-trivial complete extrinsically flat surfaces in $S^3$. In this talk, we shall introduce a representation formula for extrinsically flat surfaces in $S^3$ in terms of a pair of two curves in the 2-sphere $S^2$.

The main idea for the representation formula is the correspondence between ruled surfaces in $S^3$ and curves in $\mathcal{L}(S^3)$, where $\mathcal{L}(S^3)$ is the space of oriented geodesics in $S^3$. Then we use the metric on $\mathcal{L}(S^3)$ associated to the minitwistor complex structure.

We shall also introduce some related results for anti-de Sitter 3-space $H^3_1$.

References

Non-null Helicoidal Surfaces as Non-null Bonnet Surface

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In this paper, non-null helicoidal surfaces defined by [8], [9] and timelike Bonnet surfaces classified by [10] are taken into consideration and it is shown that the non-null helicoidal surfaces in Lorentzian space forms which admit one-parameter family of isometric deformation preserving the mean curvature are non-null bonnet surfaces.

References

Singularities of the asymptotic completion of developable Möbius strips

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Let $U$ be an open domain in Euclidean two-space $\mathbb{R}^2$ and $f : U \rightarrow \mathbb{R}^3$ a $C^\infty$ map. A point $p \in U$ is called a singular point of $f$ if the Jacobi matrix of $f$ is of rank less than 2 at $p$. Let

$$F(s, u) = \gamma(s) + u\xi(s) \quad (|u| < \epsilon)$$

be a ruled Möbius strip immersed in $\mathbb{R}^3$, where $\epsilon > 0$, $\gamma(s)$ is a generating curve and $\xi(s)$ is a ruling vector field of $F$. Then, the $C^\infty$ map

$$\tilde{F}(s, u) = \gamma(s) + u\xi(s) \quad (u \in \mathbb{R})$$

is called the asymptotic completion (or a-completion) of the immersed strip $F$. It is well-known that complete and flat (i.e. zero Gaussian curvature) surfaces immersed in $\mathbb{R}^3$ are cylindrical. This fact implies that the a-completion of a developable Möbius strip (i.e. a flat ruled Möbius strip) must have singular points. Since the most generic singular points appeared on developable surfaces are cuspidal edge singularities (cf. [1]), we are interested in how often singular points other than cuspidal edge singularities appear on the a-completion of a developable Möbius strip. We have the following result:

**Proposition.** The asymptotic completion of a developable Möbius strip has at least one singular point other than cuspidal edge singularities.

A developable Möbius strip which contains a closed geodesic is called a rectifying Möbius strip. Roughly speaking, a rectifying strip can be constructed from an isometric deformation of a rectangular domain on a plane. We also prove the following assertion:

**Theorem.** The asymptotic completion of a rectifying Möbius strip has at least three singular points other than cuspidal edge singularities.

These lower bounds of the numbers of non-cuspidal-edge singularities in the proposition and the theorem are both sharp.

**References**

Second-order Lagrangians admitting a first-order Hamiltonian formalism

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Let \( p: E \to N \) be an arbitrary fibred manifold over a connected \( n \)-dimensional manifold \( N \) oriented by a volume form \( v = dx^1 \wedge \cdots \wedge dx^n \), and let \( p^k: J^k E \to N \) be the bundle of \( k \)-jets of local sections of \( p \), with projections \( p^k_l: J^k E \to J^l E \) for every \( k \geq l \). Every fibred coordinate system \( (x^j, y^\alpha) \) on \( E \) for the projection \( p \), \( 1 \leq j \leq n \), \( 1 \leq \alpha \leq m = \dim E - \dim N \), induces a coordinate system \( (x^j, y^\alpha_I) \), on the \( r \)-jet bundle, where \( I = (i_1, \ldots, i_n) \in \mathbb{N}^n \) is an integer multi-index of order \( |I| = i_1 + \cdots + i_n \leq r \); namely,

\[
y^\alpha_I(j^r s) = \frac{\partial^{\vert I\vert} (y^\alpha \circ s)}{\partial (x^1)^{i_1} \cdots \partial (x^n)^{i_n}}(x),
\]

where \( s \) is a local section of \( p \) defined on a neighbourhood of \( x \in N \). We use the notations \( I = (j) = (0, \ldots, 0, 1, 0, \ldots, 0) \in \mathbb{N}^n \) and \( y^\alpha_{(j)} = y^\alpha_j \).

The Legendre form of a second-order Lagrangian density \( \Lambda = Lv \) defined on \( p: E \to N \), where \( L \in C^\infty(J^2 E) \), is the \( V^* (p^1) \)-valued \( p^3 \)-horizontal \( (n - 1) \)-form \( \omega_\Lambda \) on \( J^3 E \) is locally given by (e.g., see [3, 5]),

\[
\omega_\Lambda = (-1)^{i-1} L^0_i v_i \otimes dy^\alpha + (-1)^{i-1} L^i_\alpha v_i \otimes dy^\alpha_i,
\]

where \( v_i = dx^1 \wedge \cdots \wedge \widehat{dx^i} \wedge \cdots \wedge dx^n \), and

\[
L^i_\alpha = \frac{1}{2 - \delta_{ij}} \frac{\partial L}{\partial (y^\alpha_{(ij)})}, \quad (1)
\]

\[
L^0_\alpha = \frac{\partial L}{\partial y^\alpha_i} - \frac{1}{2 - \delta_{ij}} D_j \left( \frac{\partial L}{\partial (y^\alpha_{(ij)})} \right), \quad (2)
\]

where \( D_j \) denotes the “total derivative ”with respect to the coordinate \( x^j \), i.e.,

\[
D_j = \frac{\partial}{\partial x^j} + \sum_{\vert I\vert = 0}^{\infty} \sum_{\alpha = 1}^{m} y^\alpha_I(j) \frac{\partial}{\partial y^\alpha_I}.
\]

The Poincaré-Cartan form attached to \( \Lambda \) is then defined to be the ordinary \( n \)-form on \( J^3 E \) given by (e.g., see [3, 5]),

\[
\Theta_\Lambda = (p^3_2)^* \theta^2 \wedge \omega_\Lambda + \Lambda,
\]
where $\theta^2$ is the second-order structure form on $J^2E$ locally given in coordinates as follows (cf. [2], [4]):

$$
\theta^2 = \left( dy^a - y^a_i dx^i \right) \otimes \frac{\partial}{\partial y^a} + \left( dy^a_h - y^a_{(hi)} dx^i \right) \otimes \frac{\partial}{\partial y^a_h},
$$

and the exterior product of $(p_2^2)^*\theta^2$ and the Legendre form, is taken with respect to the pairing induced by duality, $V(p^1) \times_{J^1E} V^*(p^1) \rightarrow \mathbb{R}$.

The most outstanding difference with a first-order Lagrangian density is that the Legendre and Poincaré-Cartan forms associated with a second-order Lagrangian density are generally defined on $J^3E$, thus increasing by one the order of the Lagrangian density $\Lambda$.

For certain second-order Lagrangian densities it is well known that the Poincaré-Cartan form is projectable onto $J^2E$; e.g., see [1]. More precisely, the Poincaré-Cartan form of a second-order Lagrangian projects onto $J^2E$ if and only if the following system of PDEs holds (cf. [1]):

$$
\frac{1}{2-\delta_{ib}} \frac{\partial^2 L}{\partial y^\beta_{ac} \partial y^\alpha_{ib}} + \frac{1}{2-\delta_{ia}} \frac{\partial^2 L}{\partial y^\beta_{bc} \partial y^\alpha_{ia}} + \frac{1}{2-\delta_{ic}} \frac{\partial^2 L}{\partial y^\beta_{ab} \partial y^\alpha_{ic}} = 0,
$$

for all indices $1 \leq a \leq b \leq c \leq n, \alpha, \beta = 1, \ldots, m$.

More surprisingly, there exist second-order Lagrangians for which the associated Poincaré-Cartan form projects not only on $J^2E$ but also on $J^1E$. Notably, this is the case of the Einstein-Hilbert Lagrange in General Relativity.

In this talk we obtain a characterization of such Lagrangians and we study its Hamiltonian formalism.

References


Biharmonic submanifolds in Euclidean spheres

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Biharmonic maps between Riemannian manifolds represent a natural generalization of harmonic maps. In particular, a Riemannian immersion into a Euclidean space is a biharmonic map if and only if it is biharmonic in the sense of Chen. In this talk we shall survey old and new results on biharmonic submanifolds in Euclidean spheres. We shall present classification results for biharmonic hypersurfaces (according to the number of distinct principal curvatures) and biharmonic submanifolds with parallel mean curvature vector field.

Hypersurfaces in non-flat Lorentzian space forms satisfying $L_k \psi = A \psi + b$

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We study hypersurfaces either in the De Sitter space $S^{n+1}_1 \subset \mathbb{R}^{n+2}_1$ or in the anti De Sitter space $H^{n+1}_1 \subset \mathbb{R}^{n+2}_1$ whose position vector $\psi$ satisfies the condition $L_k \psi = A \psi + b$, where $L_k$ is the linearized operator of the $(k+1)$th mean curvature of the hypersurface, for a fixed $k = 0, \ldots, n-1$, $A \in \mathbb{R}^{(n+2) \times (n+2)}$ is a constant matrix and $b \in \mathbb{R}^{n+2}$ is a constant vector. For every $k$, we prove that when $A$ is self-adjoint and $b = 0$, the only hypersurfaces satisfying that condition are hypersurfaces with zero $(k+1)$-th mean curvature and constant $k$-th mean curvature, open pieces of standard pseudo-Riemannian products in $S^{n+1}_1(S^m_1(r) \times S^{n-m}(\sqrt{1+r^2}), H^m(-r) \times S^{n-m}(\sqrt{1+r^2})$, $S^m_1(\sqrt{1-r^2}) \times S^{n-m}(r)$, $H^m(-\sqrt{r^2-1}) \times S^{n-m}(r)$) open pieces of standard pseudo-Riemannian products in $H^{n+1}_1(H^m(-r) \times S^{n-m}(\sqrt{r^2-1})$, $H^m(-\sqrt{1+r^2}) \times S^{n-m}_1(r)$, $S^m_1(\sqrt{r^2-1}) \times H^{n-m}(-r)$, $H^m(-\sqrt{1-r^2}) \times H^{n-m}(-r)$) and open pieces of a quadratic hypersurfaces $\{x \in M^{n+1}_c : \langle Rx, x \rangle = d\}$ where $R$ is a self-adjoint constant matrix whose minimal polynomial is $t^2 + at + b$, with $a^2 - 4b \leq 0$, and $M^{n+1}_c$ stands for $S^{n+1}_1 \subset \mathbb{R}^{n+2}_1$ or $H^{n+1}_1 \subset \mathbb{R}^{n+2}_2$. When $H_k$ is constant and $b$ is a non-zero constant vector, we show that the hypersurface is totally umbilical, and then we also obtain a classification result.
In this study, the motion planning problem for the rigid body systems is formulated as an optimal control problem on the Lie group SO$(2, 1)$ for timelike, lightlike and spacelike cases, where the cost function to be minimized is the integral of the curvature squared. The coordinate free Maximum Principle is then applied to solve this problem. The emphasis of this study is placed on an integrable case where the necessary conditions for optimality can be analytically. In addition the corresponding optimal motions are expressed in a coordinate free manner, that is they are described completely in terms of the geometrically invariant natural curvatures. These optimal motions are shown to trace helical paths which could be useful in motion interpolation schemes. This problem formulation is both practical for the path planning application considered and illuminates how the general theory of optimal control, framed curves and left- invariant Hamiltonian systems applies to this particular setting.
A Lorentz metric on the manifold of positive definite $(2 \times 2)$-matrices and foliations by ellipses

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Let $\mathcal{E}$ be the set of all ellipses in the plane centered at zero (with axes not necessarily parallel to the coordinate axes), which may be canonically identified with the manifold $S_+$ of all positive definite $(2 \times 2)$-matrices.

The group $G = GL_2^+(\mathbb{R})$ acts smoothly on $S_+$ by $g \cdot A = gAg^T$. Consider on $G$ the bi-invariant metric of signature $(2, 2)$ given at the identity by the opposite of the canonical inner product of the split quaternions, that is, such that $\langle X, X \rangle = -\det X$ for all $X \in M_2(\mathbb{R})$, and endow $\mathcal{E} \cong S_+$ with the Lorentz metric pushed down from $G$ via the canonical projection $G \to S_+$.

We use this Lorentz metric on $\mathcal{E}$ to describe all foliations of (open sets of) the pointed plane $\mathbb{R}^2 - \{0\}$ by ellipses. More precisely, we prove that a smooth curve $\gamma$ in $\mathcal{E}$ determines a foliation of an open set of the pointed plane if and only if $\gamma$ is time-like. Moreover, if the curve $\gamma$ in $\mathcal{E}$ is defined on the whole real line, $\langle \dot{\gamma}, \dot{\gamma} \rangle = -1$, and $\langle \dot{\gamma}, X \circ \gamma \rangle$ is a bounded function, then the corresponding foliation covers the whole pointed plane (here $X$ denotes the unique unit future directed $G$-invariant vector field $X$ on $\mathcal{E}$). We also provide examples of causal curves of pairwise disjoint ellipses which do not determine a foliation.

The general setting is the characterization of the foliations of a smooth manifold by submanifolds congruent to a given one by the action of a group $H$, in terms of the $H$-invariant geometry of this set of submanifolds: Let $N$ be a smooth manifold acted on smoothly by a group $H$, let $M$ be a submanifold of $N$ and $E$ the set of submanifolds of $N$ congruent to $M$ via $G$. The problem consists in describing geometrically which subsets $F$ of $E$ determine foliations of (open subsets of) $N$. The paradigm is the paper [1], where fibrations of $S^3$ by great circles are characterized in this way. See also [2] (a partial generalization of [1]) and [3], with the global foliations of $\mathbb{R}^3$ by lines, which includes a pseudo-Riemannian reformulation of the main result of [1]. In our case, $N$ is the pointed plane, $M$ is the circle, $H = G$, $E = \mathcal{E}$ and $F$ the set of trajectories of time-like curves.

References

Clifford Cohomology of hermitian manifolds

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One of the fundamental objects in the study of a smooth manifold $M$ is its bundle of exterior differential forms, $\Lambda^*M$. This is a bundle of algebras over $M$ generated at each point by the cotangent space and in which there is defined a natural first order operator, the exterior differential $d$. The corresponding fundamental object in the study of a riemannian manifold is its Clifford bundle $Cl(M)$. This is again a bundle of algebras generated by the tangent space at each point, equipped with its inner product, and in which there is another intrinsic first order operator, the Dirac operator $D$.

In ([1]) Michelsohn uses the Clifford multiplication to elaborate a detailed analysis of Kähler manifolds. For this she considers the bundle $Cl_C(M) = Cl(M) \oplus \mathbb{C}$ and a triple of parallel operators $\mathcal{L}$, $\overline{\mathcal{L}}$ and $\mathfrak{J}$ defined on it and which carry an intrinsic $\mathfrak{sl}(2)$-structure of $Cl_C(M)$. This, together with $\mathfrak{J}$, yields a decomposition

$$Cl_C(M) \equiv \bigoplus_{|p+q|\leq n} Cl^{p,q}(M).$$

Taking the hermitian analogues of the Dirac operator $D$, she obtains operators $\mathcal{D}$ and $\overline{\mathcal{D}}$ such that $\mathcal{D}^2 = \overline{\mathcal{D}}^2 = 0$, $\mathcal{D} + \overline{\mathcal{D}} = 1/2D$ and $\overline{\mathcal{D}}$ is the formal adjoint of $\mathcal{D}$. The elements in $Cl^{p,q}$, considered as forms, are generally of mixed degrees and under this assumption the operator $\mathcal{D}$ corresponds simply to $\overline{\mathcal{D}} + \partial^*$. 

In ([3]) the authors define a formally holomorphic connection over those hermitian manifolds which satisfy the third curvature condition. The expression for this connection is

$$\nabla_X = \nabla_X^{L.C.} - \frac{1}{2} J(\nabla_X^{L.C.} \mathfrak{J})$$

where $\nabla^{L.C.}$ represents the Levi-Civita connection and $X \in T_C M$. 

In this contribution we use the algebraic theory of the Clifford algebra \( \Cl_C(M) \) developed by Michelsohn and this formally holomorphic connection to obtain similar operators to \( D \) and \( \overline{D}, D^\nabla \) and \( \overline{D}^\nabla \) respectively, on certain hermitian non Kähler manifolds, and which satisfy similar properties as, for example, \( (D^\nabla)^2 = (\overline{D}^\nabla)^2 = 0 \) and \( \overline{D}^\nabla \) is the formal adjoint of \( D^\nabla \).

References


Decoupling and Exact Solutions of Einstein Equations in Almost Kähler Variables

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We prove that the Einstein equations written in almost Kähler variables have a decoupling property which allows us to construct solutions in very general forms following methods elaborated in Refs. [1]. Such generic off-diagonal metrics are determined by corresponding classes of generating and integration functions depending, in general, on all spacetime coordinates. The almost Kähler variables are important for performing deformation and A-brane quantization of Einstein gravity and elaborating noncommutative generalizations [2] and constructing quantum corrections to solutions. We study geometric criteria when generic off-diagonal solutions a) define Lorenz manifolds and satisfy the Cauchy problem; b) generate solitonic hierarchies and model geometric Ricci flows of low dimensional geometries on four dimensional spacetime manifolds. There are considered extensions of the method for constructing exact solutions in modified theories of gravity (for instance, with extra dimensions, with scaling anisotropy and/or generalized Lagrange–Finsler spaces). Finally, we provide examples and speculate on new classes of physically important solutions and discuss geometric methods of their quantization.

Social events
Social events

\section*{Tuesday, September 6}

08:15 Registration

08:45 Opening Ceremony
   Aula Magna, Facultad de Ciencias.

20:30 Rectorat University cocktail reception at
   Hospital Real building offered by the University of Granada.

\section*{Wednesday, September 7}

21:00 Social dinner at the “Carmen de los Chapiteles”,
   Camino de la Fuente del Avellano, nº 4.

\section*{Thursday, September 8}

Afternoon Guided visit to the Alhambra monument.
   Information and tickets sold by Eurocongres. The visit includes guide and bus
   service from and back to the Faculty of Sciences.
   Bus departure at 16:15 at the front door of the Science Faculty.

\section*{Friday, September 9}

18:00 Closing Ceremony
   Aula Magna, Facultad de Ciencias.
General information

Location

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