

Some uniqueness results for the solutions to the $H_R = H_L$ surface equation

ALMA L. ALBUJER

*Departamento de Matemáticas, Universidad de Córdoba, 14071 Córdoba, Spain
alma.albuje@uco.es*

ABSTRACT

Given a domain $\Omega \subseteq \mathbb{R}^2$, we consider the differential operator given by

$$Q(u) = \operatorname{div} \left(\frac{Du}{\sqrt{1 - |Du|^2}} \right) - \operatorname{div} \left(\frac{Du}{\sqrt{1 + |Du|^2}} \right),$$

where $u \in C^2(\Omega)$, and D , div and $|\cdot|$ stand for the gradient, the divergence and the Euclidean norm on \mathbb{R}^2 . We are interested in studying the solutions to the equation

$$Q(u) = 0, \tag{1}$$

satisfying $|Du| < 1$.

Spacelike surfaces in the Lorentz-Minkowski space \mathbb{L}^3 can be endowed with two different Riemannian metrics, the metric induced by the Euclidean space \mathbb{R}^3 and the metric inherited from the Lorentz-Minkowski space \mathbb{L}^3 . Consequently, we can consider two different mean curvature functions on a spacelike surface, H_R and H_L .

On the other hand, any spacelike surface can be locally described as a spacelike graph over a domain $\Omega \subseteq \mathbb{R}^2$. Let Σ_u be the spacelike graph determined by the function u . It is easy to check that if Σ_u satisfies $H_R = H_L$, then u is a solution of (1) with $|Du| < 1$. For this reason we will refer to (1) as the $H_R = H_L$ surface equation. This equation is a quasilinear elliptic partial differential equation, everywhere except at those points at which Du vanishes, where the equation is parabolic.

In this talk we will show some uniqueness results for entire solutions to the $H_R = H_L$ surface equation, as well as for the Dirichlet problem related to it.

The results presented in this talk are part of a joint work with Magdalena Caballero [1], and with Magdalena Caballero and Enrique Sánchez [2].

References

- [1] A. L. Albuje and M. Caballero, *Geometric properties of surfaces with the same mean curvature in \mathbb{R}^3 and \mathbb{L}^3* , preprint.
- [2] A. L. Albuje, M. Caballero and E. Sánchez, *Some uniqueness results for entire solutions to the $H_R = H_L$ surface equation*, preprint.