On causality in low regularity

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ABSTRACT

The solution theory for Einstein's equations and physically relevant models of spacetimes (e.g. matched spacetimes, shock and impulsive waves, conical singularities, etc.) lead to metrics of regularity below $\mathcal{C}^{1,1}$. This class (i.e., the first derivative of the metric exists and is Lipschitz continuous) is the largest class, where the bulk of classical Lorentzian geometry remains valid. Consequently, developing Lorentzian geometry and causality with metrics below this threshold is a desirable goal for working with spacetimes of low regularity.

In this talk we will discuss causality theory for spacetimes with continuous metrics. In particular, we discuss different equivalent notions of global hyperbolicity, the causal ladder and maximal causal curves. In fact, we show that global hyperbolicity implies the existence of maximal causal curves between any two causally related points (the Avez-Seifert theorem).

Surprisingly, these maximal causal curves need not be piecewise $\mathcal{C}^1$. In [LY] it was proven that in a Riemannian manifold with α-Hölder continuous metric, geodesics (minimizing curves) are $\mathcal{C}^{1,\beta}$, where $\beta = \frac{\alpha}{2-\alpha}$. Thus an analog of [LY] cannot hold in Lorentzian geometry.

This is in part joint work with Michael Kunzinger.

References

[KS] Kunzinger M., Sämann C., *Regularity of maximal curves*, in preparation

[LY] Lytchak A., Yaman A., *On Hölder continuous Riemannian and Finsler metrics*, Trans. Amer. Math. Soc. 358 7 2006

[S] Sämann C., *Global Hyperbolicity for Spacetimes with Continuous Metrics*, to appear in Ann. Henri Poincaré, 2016