

Uniqueness of complete maximal surfaces in certain Lorentzian Warped products

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ABSTRACT

In this presentation we considered complete maximal surfaces in a Lorentzian manifold given by the warped product of the negative definite real line and a 2-dimensional surface, such that the Gauss curvature is bounded from below, and the warping function is real-positive. We characterize such surfaces satisfying a comparison involving the height function and the shape operator as slices. These estimates are observed to be optimal as we show in examples. The results are in an homonym pre-print with Alfonso Romero and Adriano Medeiros see [17].

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