

# Recent results on the oscillator spacetimes

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## ABSTRACT

*The oscillator group is a four-dimensional connected, simply connected Lie group, whose Lie algebra coincides with the one generated by the differential operators, acting on functions of one variable, associated to the harmonic oscillator problem. After its introduction [R.F. Streater, CMP 1967], the oscillator group has been extended to a one parameter family  $G_\mu$  ( $\mu > 0$ ) and proved several times to be an interesting object to study both in differential geometry and in mathematical physics.*

*The four-dimensional oscillator group is a well known homogeneous spacetime [R. Duran Diaz, P.M. Gadea and J.A. Oubiña, JMP 1999]. Its bi-invariant metric  $g_0$  was generalized to a one-parameter family  $g_a$ ,  $-1 < a < 1$ , of left-invariant Lorentzian metrics. We shall illustrate the following recent results on the geometry of Lorentzian oscillator groups:*

**Ricci solitons.** *A Ricci soliton is a pseudo-Riemannian manifold  $(M, g)$  admitting a smooth vector field  $X$ , such that*

$$\mathcal{L}_X g + \varrho = \lambda g, \tag{1}$$

*where  $\mathcal{L}_X$  and  $\varrho$  respectively denote the Lie derivative in the direction of  $X$  and the Ricci tensor and  $\lambda$  is a real number. In [1], we completely solved the system of partial differential equations, which translates (1) in a suitable set of global coordinates on  $(G_\mu, g_a)$ , proving that all these metrics are Ricci solitons (neither invariant, nor algebraic except in an extremely special case).*

**Symmetries.** *If  $(M, g)$  denotes a Lorentzian manifold and  $T$  a tensor on  $(M, g)$ , codifying some either mathematical or physical quantity, a symmetry of  $T$  is a one-parameter group of diffeomorphisms of  $(M, g)$ , leaving  $T$  invariant. Hence, it corresponds to a vector field  $X$  satisfying  $\mathcal{L}_X T = 0$ . (Examples: Killing vector fields ( $T = g$ ), curvature collineations ( $T=R$ ), Ricci collineations ( $T=\varrho$ ), matter collineations ( $T = \varrho - \frac{1}{2}\tau g$  is the energy-momentum tensor)). In [2], we obtained a complete classification of symmetries of homogeneous spacetimes  $(G_\mu, g_a)$ , also pointing out the left-invariant examples.*

## References

- [1] G. Calvaruso, *Oscillator spacetimes are Ricci solitons*, Nonlinear Analysis, to appear. DOI: 10.1016/j.na.2016.03.008.
- [2] G. Calvaruso and A. Zaeim, *On the symmetries of the Lorentzian oscillator group*, submitted.