

# Null frontal singular surfaces in Lorentzian 3-spaces

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## ABSTRACT

Let  $X$  be a 3-dimensional Lorentzian manifold (with signature  $(1, 2)$ ). A smooth map-germ  $\varphi : (\mathbf{R}^2, 0) \rightarrow X$  is called a **null frontal surface** or a **null frontal** in short if there exists a smooth lift  $\tilde{\varphi} : (\mathbf{R}^2, 0) \rightarrow PT^*X = \text{Gr}(2, TX)$  of  $\varphi$  such that  $\tilde{\varphi}(t)$  is a lightlike plane in  $T_{\varphi(t)}X$  and  $\varphi_*(T_t\mathbf{R}^2) \subset \tilde{\varphi}(t)$ , for any  $t \in (\mathbf{R}^2, 0)$ . The notion of null frontals is a natural generalization of null immersions to singular surfaces. In this talk we present several classification results of singularities which arise in null frontals up to local diffeomorphisms and up to  $O(2, 3)$ -conformal transformations in the conformally flat case (cf. [1]). The classification is achieved by using the fact that null frontals are obtained as **tangent surfaces** to null curves in  $X$ , as well as **associated varieties** to Legendre curves in the space  $Y$  of null geodesics on  $X$ . We will mention also higher dimensional cases and related topics (cf. [2][3][4]). .

## References

- [1] G. Ishikawa, Y. Machida, M. Takahashi, *Asymmetry in singularities of tangent surfaces in contact-cone Legendre-null duality*, Journal of Singularities, **3** (2011), 126–143.
- [2] G. Ishikawa, Y. Machida, M. Takahashi, *Geometry of  $D_4$  conformal triality and singularities of tangent surfaces*, Journal of Singularities, **12** (2015), 27–52.
- [3] G. Ishikawa, Y. Machida, M. Takahashi, *Singularities of tangent surfaces in Cartan’s split  $G_2$ -geometry*, Asian Journal of Mathematics, **20–2**, (2016), 353–382.
- [4] G. Ishikawa, Y. Machida, M. Takahashi,  *$D_n$ -geometry and singularities of tangent surfaces*, to appear in RIMS Kôkyûroku Bessatsu, (2016). <http://eprints3.math.sci.hokudai.ac.jp/2353/>