Lawson correspondence and Laguerre deformation

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ABSTRACT

A surface f in Euclidean space $\mathbb{R}^3$ is called L-minimal if the first variation of the surface integral $\int (H^2 - K)/K dA$ vanishes, where H and K are the mean and Gauss curvatures. The class of L-minimal surfaces is preserved by the Laguerre group which is isomorphic to the restricted Poincaré group [1]. An L-minimal f arises in general as one of the two enveloping surfaces of its Laguerre Gauss map (middle sphere congruence), which defines a spacelike immersion in Minkowski 4-space $\mathbb{R}^4_1$, away from umbilics and parabolic points. From the Laguerre Gauss map of a surface f, a Laguerre invariant quartic differential Q can be constructed, which is holomorphic if f is L-minimal [2]. A surface f is called generalized L-minimal *if the differential Q constructed from its Laguerre Gauss map is holomorphic. We show that f is generalized L-minimal if and only if it is L-minimal or is L-isothermic and locally the T-transform (Laguerre deformation) of an L-minimal isothermic surface. This is used to prove that: (1) a surface f is L-minimal isothermic if and only if its Laguerre Gauss map has zero mean curvature in some spacelike, timelike, or isotropic 3-plane of $\mathbb{R}^4_1$; and (2) the class of surfaces with holomorphic Q which are not L-minimal consists of surfaces whose Laguerre Gauss maps have constant mean curvature in some translate of hyperbolic 3-space or de Sitter 3-space in $\mathbb{R}^4_1$, or have mean curvature zero in some translate of a time-oriented lightcone in $\mathbb{R}^4_1$. As an application, we show that various instances of the Lawson isometric correspondence can be viewed as special cases of the T-transformation of L-isothermic surfaces with holomorphic quartic differential. This is based on joint work with E. Musso [3].*

References

[1] W. Blaschke: *Vorlesungen über Differentialgeometrie und geometrische Grundlagen von Einsteins Relativitätstheorie*, B. 3. Berlin: J. Springer, 1929.

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