Volume comparison for $\mathcal{C}^{1,1}$-metrics

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ABSTRACT

In recent years there has been increased interest in low regularity spacetimes due to a paper by Chruściel and Grant ([1]) showing that there exist nice approximating metrics respecting the causal structure. This sparked a variety of new results for $\mathcal{C}^{1,1}$-metrics, among them proofs that the exponential map is still a local bi-Lipschitz homeomorphism and that most results from causality theory as well as both the Hawking and the Penrose singularity theorem remain true in this regularity.
In my talk I would like to present a new result in that direction ([2]), namely a generalization of a recent volume comparison theorem for smooth Lorentzian metrics (see [3]) to this regularity. To be more precise, we shall look at globally hyperbolic spacetimes with a $\mathcal{C}^{1,1}$-metric having timelike Ricci-curvature bounded from below and establish an upper bound on the volume of (future) balls above compact subsets of smooth, spacelike, acausal and future causally complete hypersurfaces with mean curvature bounded from above. Interestingly, the proof also requires a new result regarding the measurability of the Cut locus in this regularity – generalizing a well-known fact for smooth metrics.
These volume estimates then allow us to give an alternative proof of Hawking's singularity theorem in regularity $\mathcal{C}^{1,1}$. Additionally such comparison results open the door for rigidity theorems and may even show a way to turn the tables and define Ricci curvature bounds in terms of such estimates for metrics of even lower regularity.

References

[1] P. T. Chruściel and J. D. E. Grant, *On Lorentzian causality with continuous metrics*, Classical Quantum Gravity **29** (2012).

[2] M. Graf, *Volume comparison for $\mathcal{C}^{1,1}$-metrics*, Ann. Global Anal. Geom. (2016), doi:10.1007/s10455-016-9508-2.

[3] J.-H. Treude and J. D. E. Grant, *Volume Comparison for hypersurfaces in Lorentzian manifolds and singularity theorems*, Ann. Global Anal. Geom. **43** (2013), 233–241.