Translating Solitons, Semi-Riemannian Manifolds and Lie Groups

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ABSTRACT

Famous solutions to the Mean Curvature Flow in Euclidean and Minkowski spaces are the translating solitons, which are submanifolds such that their mean curvature vector $\vec{H}$ satisfy $\vec{H} = v^{\perp}$, where $v \in \mathbb{R}^n$ is a fixed constant unit vector. For simpleness, it is very common to choose $v = (1, 0, \ldots, 0)$. These objects have been extensively studied.

Now, let (M, g) be a semi-Riemannian manifold, and $\epsilon \in \{1, -1\}$ a constant. Given a map $u : M \to \mathbb{R}$, we say that its graph $F : M \to (M \times \mathbb{R}, g + \epsilon dt^2)$ is a (vertical) translating soliton if the mean curvature vector $\vec{H}$ of F satisfies $\vec{H} = \partial_t^{\perp}$. As a first result, when the graph is semi-Riemannian, we obtain the PDE that function u must satisfy.

Next, we let a Lie group Σ act on M in such a way that the space of orbits M/Σ is diffeomorphic to an open interval $I \subset \mathbb{R}$. In this way, the PDE can be tranformed in a ODE. We are able to obtain examples. Some of them were already known, like the rotationally symmetric ones in the Euclidean and Minkowski spaces, but others are new.
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