

# Umbilical properties of spacelike co-dimension two submanifolds

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## ABSTRACT

For Riemannian submanifolds of a semi-Riemannian manifold, we introduce the concepts of *total shear tensor* and *shear operators* as the trace-free part of the corresponding second fundamental form and shape operators. The relationship between these quantities and the umbilical properties of the submanifold is shown. Several novel notions of umbilical submanifolds are then considered along with the classical concepts of totally umbilical and pseudo-umbilical submanifolds.

Then we focus on the case of co-dimension 2, and we present necessary and sufficient conditions for the submanifold to be umbilical with respect to a normal direction. Moreover, we prove that the umbilical direction, if it exists, is unique—unless the submanifold is totally umbilical—and we give a formula to compute it explicitly. When the ambient manifold is Lorentzian we also provide a way of determining its causal character. We end the paper by illustrating our results on the Lorentzian geometry of the Kerr black hole.