Existence of spacelike graphs of constant mean curvature in the steady state space

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ABSTRACT

The steady state space $\mathcal{H}^3$ *is a model for the universe proposed by Bondi, Gold and Hoyle which is homogeneous and isotropic [1, 3, 4]. From the mathematical viewpoint,* $\mathcal{H}^3$ *corresponds with the Lorentzian analogue to the upper half-space model of hyperbolic space. In this talk we consider spacelike surfaces in* $\mathcal{H}^3$ *with constant mean curvature (cmc). We study the Dirichlet problem for the prescribed mean curvature equation on a domain of a slice, that is, finding a solution* $u \in C^2(\Omega) \cap C^0(\overline{\Omega})$ *of*

$$\begin{cases} \operatorname{div}\left(\frac{Du}{\sqrt{1-|Du|^2}}\right) = \frac{2}{u}\left(H + \frac{1}{\sqrt{1-|Du|^2}}\right) & \text{in } \Omega \subset \mathbb{R}^2 \\ |Du| < 1 & \text{in } \overline{\Omega} \\ u = h & \text{along } \partial\Omega, \end{cases}$$

where $H \in \mathbb{R}$ *and* $h \in \mathbb{R}^+$*. This problem was firstly considered by Montiel in [6] for convex domains and values* $H < -1$*; see also [2] for the existence of radially symmetric spacelike cmc surfaces in* $\mathcal{H}^3$ *with different techniques. Under suitable conditions on the convexity of* Ω*, we prove the existence of spacelike cmc graphs with* $-1 \leq H < 0$ *[5]. This extends the range of* H *in the results obtained in [6]. The techniques employed are the maximum principle and the methods to establish a priori* C^1 *estimates for a cmc graph by comparing with hyperbolic planes as barriers.*

References

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