Convex functions in space-time geometry

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ABSTRACT

This is joint work with William Karr. We study classically convex and "space-time convex" functions (introduced in [1]) in space-time geometry.
Using classically convex functions, we give a geometric-topological approach to geodesic connectedness of space-times [2].
For instance: A null-disprisoning space-time is geodesically connected if it supports a proper, nonnegative strictly convex function whose critical set is a point. Timelike strictly convex hypersurfaces of Minkowski space are geodesically connected. We also give a criterion for the existence of a convex function on a semi-Riemannian manifold, and compare our results to those obtained by analytic methods (see [3]).
Turning to space-time convex functions [4], we prove they are fundamentally related to curvature bounds of the form $\mathcal{R} \leq K$ (i.e. space-like sectional curvatures $\leq K$, timelike sectional curvatures $\geq K$). These bounds were introduced and applied in [5] and characterized geometrically in [6]. The connection with curvature bounds allows us to identify many space-times that support space-time convex functions. Such functions rule out, for example, closed spacelike geodesics and closed marginally inner and outer trapped surfaces [1].

References

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