

# Geometric structures of Hamilton-Jacobi equations associated with metrics

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## ABSTRACT

In this presentation, we associate to an indefinite metric on a space  $X$  a Hamilton-Jacobi equation, and construct their complete solutions and general solutions from Legendre manifolds in the space  $Y$  of characteristic curves with a contact structure. We especially study the standard  $(1, 2)$ -metric (in the case  $B_2 = C_2$ ),  $(1, 3)$ -metric and  $(2, 2)$ -metric (in the case  $D_3 = A_3$ ), via geometric structures of double fibrations. We remark that the null direction bundle  $Z$  on  $X$  is regarded as an hypersurface of the 1-jet space  $J^1(X)$ , which defines a Hamilton-Jacobi equation intrinsically. Then the solutions are regarded as maximal integral submanifolds to the derived system of the canonical distribution on  $Z$ . By considering cone structures, we can discuss generalizations of our constructions to general Hamiltonians, second order PDEs, etc.