Induced Riemannian structures on null hypersurfaces

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ABSTRACT

A hypersurface in a Lorentzian manifold is null if the induced metric tensor is degenerate on it. These hypersurfaces do not have a Riemannian counterpart, so they are interesting by their own, both geometrically and physically. Null hypersurfaces can not be treated as spacelike or timelike hypersurfaces, since an orthogonal projection can not be defined on them. So, neither the second fundamental form or the induced connection can be constructed in the usual way and specific techniques were developed for this. One of the most usual (but not the unique) is to fix a geometric data formed by a null section and a screen distribution on the null hypersurface. This allows to define an induced connection and a null second fundamental form, which gives the expected information on the extrinsic geometry. However, the induced connection does not arise necessarily from a metric and is clear that it is not an appropriate tool to study intrinsic geometric properties. Moreover, both the null section and the screen distribution are fixed arbitrarily and independently and it is not clear how to choose them in order to have a reasonable coupling between the properties of the null hypersuperface and the ambient space.
Alternatively, we show a technique to construct a Riemannian metric $\widetilde{g}$ *on a null hypersurface* L*. It is based on the arbitrary choice of a transverse vector field, called rigging field, from which we construct a null section, which we call rigged field and a screen distribution. The improvement over the above technique is twofold: first, the geometric data depends only on the choice of a unique object, the rigging field. Secondly, we introduce a Riemannian structure coupled with it, which is used to study the null hypersurface. Those structures are not natural in the sense that they depend on the choice of the rigging field, but the flexibility to choose it turns this limitation into an advantage, allowing us to use valuable information on the ambient space, for example in the presence of symmetries.*
We relate extrinsic properties of the null hypersurface to the properties of the Riemannian manifold $(L, \widetilde{g})$ *and we establish some formulas linking the curvature of the ambient manifold and the curvature of* $(L, \widetilde{g})$*. This allows us to obtain some new results on null hypersuperfaces. For example, we use the Bochner technique to show a curvature condition which implies that a compact totally umbilic null hypersurface must be totally geodesic. We also show that the induced Riemannian metric* $\widetilde{g}$ *in a totally umbilic null hypersurface is locally a twisted product, which can be a warped or direct product depending on the properties of the ambient space and the rigging field. This is used to prove that the first conjugate point of a null geodesic contained in a totally umbilic null cone has maximum multiplicity. Finally, we adapt the main ideas to null submanifods of arbitrary codimension, which allows us to apply Gauss-Bonnet theorem to compact null surfaces.*

References

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