

# Lorentzian generalized quasi-Einstein metrics and static structures

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## ABSTRACT

*A Lorentzian manifold  $(M, g)$  is a generalized quasi-Einstein space if there is a solution  $f \in C^\infty(M)$  of the equation [2]*

$$\text{Hes}_f + \rho - \mu df \otimes df = \lambda g \quad (1)$$

*for some  $\mu \in \mathbb{R}$  and  $\lambda \in C^\infty(M)$ . Generalized quasi-Einstein metrics contain Einstein metrics, gradient Ricci solitons and gradient Ricci almost solitons as special cases. There are, however, other important situations when  $\mu \neq 0$ . In such a case, Equation (1) is equivalent to*

$$\text{Hes}_u - \mu \rho = -\mu \lambda u g \quad (2)$$

*and corresponds to conformally Einstein metrics [1] (when  $\mu = -\frac{1}{n-2}$  and  $\lambda = \frac{n-2}{n} \frac{\Delta u}{u} + \frac{\tau}{n}$ ,  $\tau$  being the scalar curvature and  $n = \dim M$ ) and to the static structures (when  $\mu = 1$  and  $\lambda = \frac{1}{n}(\tau - \frac{\Delta f}{f})$ ).*

*Our purpose is to present some new results on the classification of locally conformally flat generalized quasi-Einstein Lorentzian manifolds.*

## References

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