On the so-called “tunnelling interpretation” of black hole radiation: The algebraic QFT viewpoint

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Plan of the talk

- Hawking radiation, Killing horizons, Hadamard states
- "Tunnelling" interpretation

Re-formulation in the framework of AQFT

- Geometric and algebraic setup
- Main theorem
- ϕ^3 self-interaction: a toy model
- Summary and open issues

Talk based on

- V. M., N. Pinamonti **Commun.Math.Phys.** 309 (2012) 295-311
- G. Collini, V. M., N. Pinamonti **Lett.Math.Phys.**104 (2014) 217-232

Hawking radiation, Killing horizons and Hadamard states

- Black hole radiation with temperature $T_H = \frac{\kappa}{2\pi}$ is predicted to appear at future null infinity $\mathcal{I}^+$. In this view the complete (future) black hole structure seems to be necessary. The future **Killing event horizon** $\mathcal{H}$ in particular.
- In a spherically symmetric spacetime with collapsing matter leading to a BH, a linear KG field in a **Hadamard state** shows thermal radiation on $\mathcal{I}^+$ with temperature T_H (independently form the details of the collapse). [FredenhagenHaag90]
- Actually there is a more recent an very popular point of view supporting another intepretation of black hole radiation.

Parikh-Wilczek and Volovik's "Tunnelling" interpretation

"Tunnelling" interpretation **[Parikh -Wilczek00]**, **[Volovik99]**

Tunnelling probability P_E through a Schwarzschild BH horizon for a particle with energy E has **thermal** behaviour ($k_{Boltzmann} = 1$):

$$P_E \;\alpha\; \exp(-E/T_H)\,.$$

$\Rightarrow$ here the BH radiation is viewed as a **local** phenomenon

- no detection **at future null infinity** necessary,
- **no global BH structure** seems to be necessary: just a neighbourhood of a point on the horizon was used in computations.

Some details (disregarding all problems)

- Tunnelling probability through a **Schwarzschild** horizon of a for a *particle* with energy E:

$$\Gamma_E = |\langle \psi_{Ex} | \psi_{Ey} \rangle|^2$$

ψ_{Ex}, ψ_{Ey} localized around $x = (t, r_1, \theta, \varphi)$ and $y = (t, r_2, \theta, \varphi)$, **separated by the horizon**.

- "Quantization" w.r.to the **Painlevé time** t:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + 2\sqrt{\frac{2M}{r}}\, dt\, dr + dr^2 + r^2 d\Omega$$

- **WKB method** to approximate Γ_E:

$$\Gamma_E = \lim_{y \to x} |\langle \psi_{Ex} | \psi_{Ey} \rangle|^2 \sim \lim_{y \to x} |e^{i \int_{r_1}^{r_2} p_r^{(E)} dr}|^2$$

The integral diverges $\Rightarrow$ complex plane regularization. An imaginary part arises:

$$\Gamma_E \sim e^{-2 Im S_{reg}} \sim e^{-E/T_H}$$

Problems!

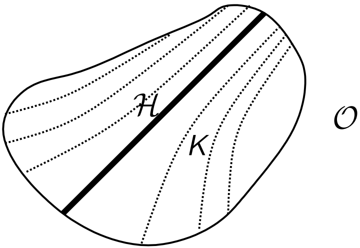
About 1000 citations by subsequent papers, performing all possible variations, without tackling the real problems of this (however very interesting) approach.

- Notions of **particle**, **time** and **energy** are ambiguously defined in **curved spacetime** (no Poincaré symmetry). **Localized** particles with **defined** energy...
- There is no here a **Schrödinger equation** to handle by means of **WKB machinery**. **KG fields** are not **Schroedinger waves**.
- Appearance of T_H suspiciously related with the choice of the complex-plane regularization procedure.

It is however difficult to suppose that T_H shows up by chance.
To investigate the issue the whole computation should be performed within the proper framework of QFT in curved spacetime.

The geometry of local Killing horizon

- From now on, we focus on some **neighbourhood** $\mathcal{O}$ of a point on the **horizon** $\mathcal{H}$ of a black hole generated by the **Killing field** K,
- Actually, since our computation will not depend on the geometry outside $\mathcal{O}$ the horizon may (smoothly) cease to exist outside $\mathcal{O}$.



Local geometric hypotheses (LGH) for $\mathcal{O}$

Here are the precise hypotheses we assume for $\mathcal{O}$.

Definition

If (M, g) is a time-orientable smooth spacetime, **LGH** hold for an open set $\mathcal{O} \subset M$, if a smooth vector field K exists thereon such that:

(a) K is a Killing field for g in $\mathcal{O}$.

(b) $\mathcal{O}$ contains the **local Killing horizon** $\mathcal{H}$ i.e. a 3-submanifold invariant under the action of K with $K^a K_a = 0$ on $\mathcal{H}$.

(c) The **orbits** of K in $\mathcal{O}$ are diffeomorphic to an **open interval** I and topologically $\mathcal{H} = I \times \mathcal{B}$ ($\mathcal{B}$ being a 2-dimensional cross section).

(d) The **surface gravity** κ is **constant** on $\mathcal{H}$ and $\kappa \neq 0$ thereon. (κ is defined by $\nabla^a(K_b K^b) = -2\kappa K^a$.)

Comments on our geometric hypotheses

- The requirement $\kappa =$ constant along $\mathcal{H}$ means that the **thermodynamical equilibrium** has been reached on $\mathcal{H}$, since $\kappa = 2\pi T_H$.

- **LGH** are in particular satisfied by neighbourhoods of points on the **future horizon** of a **non-extremal** black hole in the **Kerr-Newman family**, including **charged** and **rotating black holes**.

- **LGH** are valid for "realistic" black holes produced by collapsed matter, so that only the future horizon exists, but even for **eternal black holes** (whose manifolds include **white hole** regions as in the whole Kruskal spacetime)
- Our picture includes also situations where the collapse starts, reaches a sort of local equilibrium and it **stops** after a while, without giving rise to a complete BH structure.

The algebra of quantum fields

- $\mathcal{A}$ is the unital $*$-**algebra** generated by the linear abstract **field operators** $\phi(f)$ with $f \in C_0^\infty(M)$ such that:
  - $\phi(f)^* = \phi(\overline{f})$
  - $[\phi(f), \phi(f')] = 0$ for causally disjoint $supp(f)$, $supp(f')$.

  No field equation is necessary.

- We intend to compute the **correlation function** $\omega(\phi(f)\phi(f'))$ when $supp(f), supp(f') \subset \mathcal{O}$ are "very close" to $\mathcal{H}$.
- When $supp(f), supp(f')$ are separated by the horizon, up to the normalization, $|\omega(\phi(f)\phi(f'))|^2$ should correspond to the **"tunnelling probability"** of PW (representing everyting in a Hilbert space via the GNS theorem).

  Hypothesis on the states: Supposing that $\mathcal{O}$ is **geodesically convex** we assume that:

$$\omega_2(x, y) := \frac{U(x, y)}{\sigma_\epsilon(x, y)} + w_\epsilon(x, y)$$

The algebra of quantum fields

- if t is a time coordinate increasing toward the future: $\sigma_\epsilon(x,y) = \sigma(x,y) + 2i\epsilon(t_x - t_y) + \epsilon$
- There exists $c > 0$ constant such that $U(x,y) = c$ if $x, y \in \mathcal{H}$ and $\sigma(x,y) = 0$.
- $w_\epsilon(p,p')$ bi-distribution less singular than $1/\sigma_\epsilon$.

  (i) $w_\epsilon(p,p') \to w'(p,p')$, a.e. in (p,p') as $\epsilon \to 0^+$
  w_ϵ is ϵ-unif. bounded by loc. $\mathcal{O}^2$-integrable function;

  (ii) $w'(V,U,s,V',U',s') \to w''(U,s,U',s')$ a.e. in (U,s,U',s') as $(V,V') \to (0,0)$,
  w' is (V,V')-unif. bounded by loc. $\mathcal{H}^2$-integrable function.

EXAMPLE: $H(x,y)$ Hadamard parametrix, $\delta > 0$, and f, g, h smooth

$$\omega_2(x,y) = H(x,y) + \frac{f(x,y)}{\sigma_\epsilon^{1-\delta}(x,y)} + g(x,y)\ln\sigma_\epsilon + h(x,y)$$

Shrinking procedure for smearing functions

Coordinates (U, V, x^3, x^4) in a neighbourhood of $\mathcal{H}$

- x^3,x^4 coordinates on a cross section $\mathcal{B}$.
- U affine parameter of null geodesics forming $\mathcal{H}$
- V affine parameter of null geodesics crossing $\mathcal{H}$ $(V = 0)$

Re-interpreting "$x \to y$" in terms of test functions f, f', we compute:

$$\lim_{\lambda \to 0^+} |\omega(\phi(f_\lambda)\phi(f'_\lambda))|^2$$

up to normalization, one-particle state transition probability if ω Gaussian
The limit $\lambda \to 0^+$ **shrinks the supports of f and f' towards $\mathcal{H}$**:

$$f_\lambda(V, U, x) = \frac{1}{\lambda} f\left(\frac{V}{\lambda}, U, x\right)$$

To remove a divergence $(V \sim 0)$ we assume

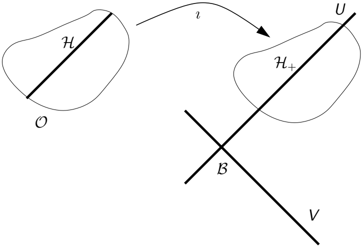
$$f = \partial_V F\ , \qquad f' = \partial_V F'\ , \quad F, F' \in C_0^\infty(\mathcal{O}),$$

just a technical geometric detail

A important technical step is given by the following result.

Theorem

(Racz and Wald [1992 1996])
Let $\mathcal{O} \subset M$ satisfy the LGH. There is a time-orientable spacetime (M', g') with a **bifurcate Killing horizon** *generated by a Killing field K' and an isometric imbedding $i : \mathcal{O} \to M'$ such that $i_*(K) = K'$.*



$\mathcal{H}$ becomes a part of a **future Killing horizon** $\mathcal{H}_+$ in the future of the **bifurcation surface** $\mathcal{B}$, where $K' = 0$.

Main Theorem

Extending technical results of [Kay-Wald91], exploiting [Racz-Wald92 96]

Theorem (1)

Assuming that **LGH** *hold in* $\mathcal{O}$ *and that*

$$\omega_2(x,y) = \frac{U(x,y)}{\sigma(x,y)} + \textit{less singular terms}\,,$$

for f *and* f' *with* **supports separated by** $\mathcal{H}$*, it holds:*

$$\lim_{\lambda \to 0^+} \omega\left(\phi(f_\lambda)\phi(f'_\lambda)\right) =$$

$$= -\lim_{\epsilon \to 0^+} \int_{\mathbb{R}^4 \times \mathcal{B}} \frac{F(V,U,s)F'(V',U',s)}{16\pi(V - V' - i\epsilon)^2}\, dUdVdU'dV'd\mu(s)\,.$$

μ *being the measure induced by* h *on* $\mathcal{B}$*.*

Consequences: thermal behaviour

- Natural notion of time (for the external region at least): **the parameter τ of the Killing field K integral lines.**

$$V(\tau) \simeq -e^{-\kappa\tau} \text{ "int. region"} \qquad V(\tau) \simeq e^{-\kappa\tau}\text{, "ext. region"}$$

- The τ-**Fourier transform** $\hat{F}(E)$ of $F(\tau)$ defines the **energy spectrum** referred to the notion of energy E associated with τ.

Corollary

$$\omega(\phi(f_\lambda)\phi(f'_\lambda))|_{\lambda=0} = \int_{\mathbb{R}^2\times\mathcal{B}} \int_{-\infty}^{\infty} \frac{\overline{\widehat{F}(E,U,x)}\widehat{F'}(E,U',x)}{16\sinh(\beta_H E/2)} E dE \, dU dU' d\mu(x)\,.$$

$$\Rightarrow \quad \lim_{\lambda\to 0} |\omega(\phi(f_\lambda)\phi(f'_\lambda))|^2 \sim C\, E_0^2\, e^{-\beta_H E_0}\,,$$

for f, f' separated by $\mathcal{H}$ with energy concentrated around $E_0 > 0$.

Consequences: local thermal behaviour

REMARKS

- The found estimate holds also taking normalization of states into account (probability transition): C changes remaining independent from $\beta_H = 1/T_H$.

- The result is completely local however a physical ambiguity exists. K can be rescaled with a constant $c > 0$ since $cK(x)$ remains a Killing vector determining the same Killing Horizon, however changing the temperature $T_H \to cT_H$.
  The value of K has to be fixed somewhere (at space infinity in an asymptotically flat black hole spacetime, requiring $K \to \partial_{t_{\text{Minkowski}}}$).
- However the local temperature measured by a K-stationary thermometer it is not affected by the ambiguity since it includes the Tolman red-shift factor:

$$T(x) = T_H/(-K_a(x)K^a(x))^{1/2}$$

The simplest interacting model: ϕ^3 in Rindler spacetime.1

- It is not obvious whether the result holds **perturbatively** taking **renormalization** into account for an **interacting** theory.
- Simplest case: **one-loop** computation for $\mathcal{L}_I = \frac{g}{3!}{:}\phi^3{:}$ referring to Killing **Rindler horizon** in **Minkowski spacetime** and **Poincaré invariant vacuum**. Do the hypotheses of **Theorem 1** hold for $\langle \Psi, \phi(x)\phi(y)\Psi\rangle$? (**NB**: Ψ interacting vacuum, ϕ interacting field)
- The hope is that the result extends to curved spacetime for **Hadamard states** exploiting the whole **generally locally covariant** renormalization procedure (BrunettiFredenhagen Hollands-Wald)
- We need a **detailed expression** for the integral kernel:

$$\langle \Psi, T[\phi(x)\phi(y)]\Psi\rangle = \frac{\langle \Psi_0, T[\phi_0(x)\phi_0(y)S]\Psi_0\rangle}{\langle \Psi_0, TS(g)\Psi_0\rangle}$$

$$S = I + \frac{ig}{3!}\int_M {:}\phi_0^3{:}(u)d^4u - \frac{g^2}{2!(3!)^2}\int_{M^2} {:}\phi_0^3{:}(u){:}\phi_0^3{:}(u')d^4ud^4u' + \cdots .$$

The simplest interacting model: ϕ^3 in Rindler spacetime.2

- Notice that for x and y **separated by** $\mathcal{H}$:

$$\langle \Psi, \phi(x)\phi(y)\Psi\rangle = \langle \Psi, T[\phi(x)\phi(y)]\Psi\rangle$$

- Adopting the *Epstein-Glaser renormalization procedure:*

$$\langle \Psi, \phi(x)\phi(y)\Psi\rangle = -iG_F(x,y) + A\int_M G_F(x,u)G_F(u,y)d^4u$$

$$-\frac{g^2}{2!}\int_{M^2} G_F(x,u)G_F^{2(ext)}(u,u')G_F(z',y)d^4ud^4u' + \cdots$$

A is a **finite renorm.const.**, *G_F the free Feynman propagator and $\widehat{G_F^{2(ext)}}(k)$ is an "extension to the diagonal" of $(\widehat{G_F}(k))^2$ given by:*

$$= \frac{i}{2(2\pi)^4}\left[-1+\sqrt{1+\frac{4m^2-i\epsilon}{k^2}}\coth^{-1}\left(\sqrt{1+\frac{4m^2-i\epsilon}{k^2}}\right)\right].$$

- *residue calculus difficult due to the contribution of* **branch cuts**.

The simplest interacting model: ϕ^3 in Rindler spacetime.3

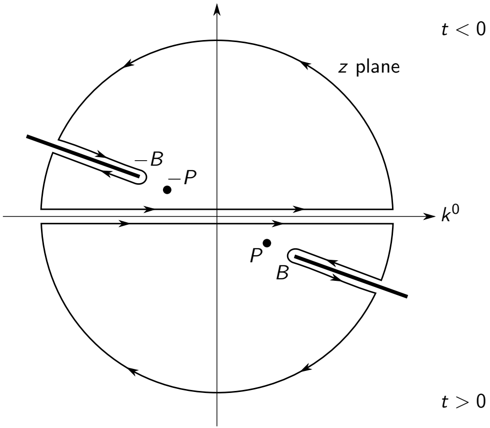


Figure: $\widehat{G_F^{2(ext)}}((z, \mathbf{k}))$. Poles at $\pm P = \pm\sqrt{\mathbf{k}^2 + m^2 - i\epsilon}$ and branch cuts starting at $\pm B = \pm\sqrt{\mathbf{k}^2 + 4m^2 - i\epsilon}$

The simplest interacting model: ϕ^3 in Rindler spacetime.4

- The final result is:

$$\langle \Psi, \phi(x)\phi(y)\Psi \rangle = AK_0\left(\sqrt{m^2\sigma_\epsilon(x,y)}\right) +$$

$$\left[\frac{m^2}{(2\pi)^2} - \frac{g^2}{4(2\pi)^2}\left(\frac{1}{2} - \frac{\pi}{3\sqrt{3}}\right)\right]\frac{K_1\left(\sqrt{m^2\sigma_\epsilon(x,y)}\right)}{\sqrt{m^2\sigma_\epsilon(x,y)}} + \mathcal{K}(\sigma_\epsilon(x,y)) + \ldots$$

where $\sigma_\epsilon(x,y) := \sigma(x,y) + 2i\epsilon(t_x - t_y) + \epsilon^2$ and

$$\mathcal{K}(u) := \frac{g^2}{4(2\pi)^4}\frac{1}{\sqrt{u}}\int_{2m}^{+\infty} dM \frac{\sqrt{M^2 - 4m^2}}{(M^2 - m^2)^2} MK_1\left(M\sqrt{u}\right) .$$

- A **short-distance** analysis shows that, for suitable functions $f_\epsilon, h_\epsilon, r_\epsilon$:

$$\langle \Psi, \phi(x)\phi(y)\Psi \rangle = \frac{1}{(2\pi)^2\sigma_\epsilon} + \frac{h(x,y)}{\sqrt{\sigma_\epsilon(x,y)}} + f(x,y)\ln\sigma_\epsilon(x,y) + r(x,y) ,$$

The simplest interacting model: ϕ^3 in Rindler spacetime.5

Since the hypotheses of Theorem 1 are valid, the conclusion is that:

In Minkowski spacetime, even taking the **one-loop radiative corrections** *of the interaction* $\mathcal{L}_I = \frac{g}{3!}\phi^3$ *into account, i.e. referring to the one-loop* **renormalized vacuum state** Ψ *and* **renormalized field operators** ϕ*:*

$$\lim_{\lambda\to 0} |\langle\Psi, \phi(f_\lambda)\phi(f'_\lambda)\Psi\rangle|^2 \sim C\, E_0^2\, e^{-\beta_H E_0}\,,$$

for packets concentrated around E_0 *of the (Killing) energy.*

Comments and open issues. 1

- We have proved that Parikh-Wilczek and Volovik's result has a **precise** and **rigorous** meaning if adopting the viewpoint of algebraic QFT in curved spacetime.
- Our computation of the "tunnelling probability" is **local in space and time**, and it strongly supports the idea that the **Hawking radiation is (also) a local phenomenon**, independent from the existence of a whole black hole. However our results work for the full Kerr-Newman class of non-extreme black holes, including the **charged rotating** one, eternal or produced by a collapse.
- The results are **independent from any field equation** and are valid for a class of states including **Hadamard states**.

Comments, further developments, open issues. 2

- We expect that the results may be valid (for ϕ^3) also in curved spacetime starting from **Hadamard states**, since the simplest model, ϕ^3 in Minkowski spacetime supports that idea. (Very difficult computations,... task for PhD students)

- For the ϕ^n model, the **divergence structure** of the two point function is should be of the form (*S.Hollands' remark):*

$$\langle\Psi, \phi(x)\phi(y)\Psi\rangle = \frac{a_0}{\sigma} + \sum_{k=1}^{N} a_k \frac{(\ln\sigma)^k}{\sigma} + \textit{more reg. terms}\,,$$

Results are preserved if the factor of rescaling procedure is corrected:

$$\frac{1}{\lambda} \to \frac{1}{\lambda(\ln|\lambda|)^{N/2}}$$

Possible problem: N may depend on the perturbative order.

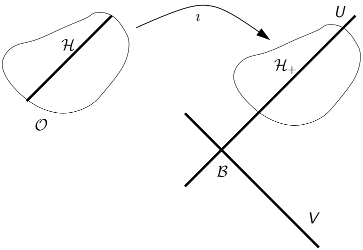
Thanks a lot for your attention!

Adding the bifurcation surface $\mathcal{B}$

An important technical step is given by the following result.

Theorem

(Racz and Wald [1992 1996])
Let $\mathcal{O} \subset M$ satisfy the LGH. There is a time-orientable spacetime (M', g') with a **bifurcate Killing horizon** *generated by a Killing field K' and an isometric imbedding $i : \mathcal{O} \to M'$ such that $i_*(K) = K'$.*



$\mathcal{H}$ becomes a part of a **future Killing horizon** $\mathcal{H}_+$ in the future of the **bifurcation surface** $\mathcal{B}$, where $K' = 0$.

Coordinate patch adapted to $\mathcal{H}_+$, using $\mathcal{B}$

- In a neighbourhood of $\mathcal{H}_+$ we can define coordinates (U, V, x^3, x^4)
  - x^3,x^4 coordinates on $\mathcal{B}$.
  - U affine parameter along null geodesics forming $\mathcal{H}_+$ ($U = 0$ on $\mathcal{B}$)
  - V affine parameter along null geodesics crossing $\mathcal{H}_+$ ($V = 0$ on $\mathcal{H}_+$)
- Exploiting that coordinate frame with $g(\partial_V, \partial_U) = -1$ on $\mathcal{H}_+$, the metric on $\mathcal{H}$ in $\mathcal{O}$ takes the form:

$$g\restriction_{\mathcal{H}} = -\frac{1}{2}dU \otimes dV - \frac{1}{2}dV \otimes dU + \sum_{i,j=3}^{4} h_{ij}(x^3, x^4)dx^i \otimes dx^j ,$$

- h – independent from V, U – is the **Euclidean** metric on the bifurcation surface $\mathcal{B}$: a **spacelike** 2-**surface**.
- In $\mathcal{O}$ the Killing field K takes the form:

$$K^V(p) = -\kappa V + V^2 R_1(p) , \quad K^U(p) = \kappa U + V^2 R_2(p) ,$$

$$K^i(p) = VR_i(p) , \ i = 3, 4 ,$$

R_1, R_2, R_i bounded smooth functions.

Some technical comments

- Result similar to that by Fredenhagen and Haag but now it holds for a generic local Killing horizon and no spherical symmetry is required.
- The proof is mainly based on the following expression for $\sigma(p,q)$ appearing in $\omega_2(p,q)$. Let $s : \mathcal{O} \to \mathcal{B}$ be the natural projection onto $\mathcal{B}$, if $p, q \in \mathcal{O}$:

$$\sigma(p,q) = \ell(s(p), s(q)) - (U_p - U_q)(V_p - V_q) + R(p, V_q, U_q)$$

  ℓ squared **geodesic distance** on $(\mathcal{B}, h)$,
  $R(p, V_q, U_q) = AV_p^2 + BV_q^2 + CV_pV_q$,
  A, B, C bounded smooth functions of p, V_q, U_q.
  It generalizes a similar technical result found and used by Kay and Wald.

Source of the Hawking radiation

Case of f_λ and f'_λ with both supports in the external region.

$$\omega(\phi(f_\lambda)\phi(f'_\lambda))|_{\lambda=0} = -\lim_{\epsilon\to 0^+}\int \frac{\kappa^2 F(\tau,U,x)F'(\tau',U',x)}{64\pi(\sinh(\frac{\kappa}{2}(\tau-\tau'))+i\epsilon)^2} d\tau dU d\tau' dU' d\mu$$

The τ-**Fourier transf.** produces the **Bose spectrum** at the Hawking temperature:

$$\omega(\Phi(f_\lambda)\Phi(f'_\lambda))|_{\lambda=0} = \int_{\mathbb{R}^2\times\mathcal{B}} \int_{-\infty}^{\infty} \frac{\overline{\widehat{F}(E,U,x)}\widehat{F'}(E,U',x)}{1-e^{-\beta_H E}} E dE \frac{dU dU' d\mu(x)}{32},$$

That result generalises the core of the Fredenhagen-Haag's explanation of the Hawking radiation for $\mathcal{H}$ complete in the future (in the spherically symmetric case and exploiting Klein-Gordon equation).