XXIII International Fall Workshop on Geometry and Physics

2 to 5 September 2014, Granada, Spain

Book of abstracts

Schedule

	Tuesday 2nd	Wednesday 3rd	Thursday 4th	Friday 5th
9:00-10:00		Ercolessi (Course 1)	Ercolessi (Course 1)	Ercolessi (Course 1)
10:00-11:00		Martín (Course 2)	Martín (Course 2)	Martín (Course 2)
11:00-11:30		Coffee	Coffee (+ poll for 2 posters)	Coffee
11:30-12:30		Berndt	Hollands	Dafermos
12:30-13:00		Ferreira	Marathe	Aldaya (+ photo)
13:00-13:30		Bartolo	Villacampa	
	Opening 15:00	Lunch	Lunch	Lunch
15:15-16:15	Diacu	Moretti	Talk Alhambra (14:30-15:30) Bus Alhambra (15:30)	Voted Talk 1 Voted Talk 2
16:15-16:45	Santos	Pastorello		Alarcón
16:45-17:30	Coffee+Posters	Coffee +Posters		Coffee+Posters (until 17:15)
17:30-18:00	Huerta	Grabowska		Pérez-Pardo (17:15-18:15)
18:00-19:00	Enciso	Fathi		
			Reception at Carmen de los Mártires (20:30)	

I. Minicourses

Methods of quantization

Elisa Ercolessi

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Abstract

This course aims at giving an introduction to some modern methods of quantization. More specifically we will deal with:

– Weyl quantization on phase space: exponential form of canonical commutation relations, properties of Weyl operators, Moyal product.

– Coherent States and Bargmann-Fock representation: properties of coherent states, Fock basis, the complex phase space representation.

– Feynman's Path Integral: Green kernel of the Schrödinger operator, path integral approach in phase space, in coordinate space, in coherent space representation.

Throughout the three lectures, the free particle, the harmonic oscillator and the two-level system will be used as guide examples to see the main principles in action.

Mean curvature flow and related topics

Francisco Martín

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Abstract

The aim of our lectures is giving a brief idea about singularity formation, non-uniqueness and topological change under motion by mean curvature. Mean curvature flow arises as a simplified model in several physical problems where surface tension plays a role.

Contents

I. Mean curvature flows in Euclidean space.

- I.1. Introduction.
- I.2. Existence.
- I.3. Geometric evolution equations.
- I.4. Comparison principle.
- I.5. Examples.

II. Singularities and solitons.

- II.1. Types of singularities.
- II.2. Integral estimates and monotonicity formulas.
- II.3. Solitons.

References

[1] K. Ecker: *Regularity Theory for Mean Curvature Flow*. Birkhäuser, 2004.

[2] C. Mantegazza: *Lecture Notes on Mean Curvature Flow*. Birkhäuser, 2011.

[3] F. Martín, J. Pérez: *Lecture notes on Mean curvature flow and related topics*, 2014. To be delivered at the meeting.

[4] M. Ritoré, C. Sinestrari: *Mean Curvature Flow and Isoperimetric Inequalities*. Birkhäuser, 2010.

[5] K. Smoczyk: *Mean Curvature Flow in higher codimension. Introduction and survey.* Preprint, arXiv:1104.3222 [math.DG].

II. Invited lectures

Symmetries of non-linear systems and non-canonical quantization

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Abstract

We face a sound revision of the role of symmetries of a physical system so as to characterize the corresponding solution manifold by means of Noether invariants. This step is inescapable to achieve the correct quantization in non-linear cases, where the success of Canonical Quantization is not guaranteed. To this end, point symmetries of the Lagrangian are generally not enough, and the generalized use of contact symmetries will play a preponderant tool. They are defined in terms of the Poincaré-Cartan form, which permits us to find the symplectic structure on the Solution Manifold, through some sort of Hamilton-Jacobi transformation, as well as the required symmetries. They are realized as Hamiltonian vector fields, associated with functions on the solution manifold, lifted back to the Evolution Manifold through the inverse of this Hamilton-Jacobi mapping, that which constitutes an inverse of the Noether Theorem. In this framework, solutions and symmetries are somehow identified and this correspondence is also kept at a well-defined perturbative level. The next step in approaching the quantization consists in selecting the proper Poisson subalgebra to replace the standard Heisenberg-Weyl one, and proceeding to construct their unitary and irreducible representations according to an already well-established group-theoretical method. The paradigmatic example of Non-Linear Sigma Models will be considered in the context of Non-Abelian Stueckelberg approach to the Quantum Field Theory of Massive Gauge Theories.

Polar actions on symmetric spaces

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Abstract

An isometric action of a connected Lie group on a Riemannian manifold is called polar if there exists a connected closed submanifold that meets each orbit of the action and intersects it orthogonally. Dadok established in 1985 a remarkable, and mysterious, relation between polar actions on Euclidean spaces and Riemannian symmetric spaces. Soon afterwards an attempt was made to classify polar actions on symmetric spaces. For irreducible symmetric spaces of compact type the final step of the classification has recently been achieved by Kollross and Lytchak. In the talk I want to focus on symmetric spaces of noncompact type. For actions of reductive groups one can use the concept of duality between symmetric spaces of compact type and of noncompact type. However, new examples and phenomena arise from the geometry induced by actions of parabolic subgroups, for which there is no analogue in the compact case. I plan to discuss the main difficulties one encounters here and some partial solutions.

The mathematical analysis of black hole spacetimes in general relativity

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Newton's equations in spaces of constant curvature

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Abstract

We consider a natural extension of Newton's equations of the N-body problem to spaces of constant curvature. We first present some qualitative results regarding the motion of the bodies, focusing on relative equilibria and rotopulsators, which generalize the notion of homographic orbits from Euclidean to curved space. Then we write the equations in intrinsic coordinates and discuss the advantages and disadvantages of this approach. Finally we come up with a new and simple form of the equations that brings together the Euclidean case (of Gaussian curvature $k = 0$), the hyperbolic case (of Gaussian curvature $k < 0$), and the elliptic case (of Gaussian curvature $k > 0$). Thus Newton's classical equations can be regarded in a broader context, namely that in which the motion of the bodies takes place in spaces of constant curvature. The equations of motion depend on the curvature k, and the Euclidean case is recovered when $k = 0$. This conclusion could not be analytically drawn from previously known forms of the equations of motion in curved space since taking $k \to 0$, for both $k > 0$ and $k < 0$, led to undetermined expressions, although it was geometrically and mechanically clear that the Newtonian equations are recovered in the limit. This new form of the equations of motion allows the study of the classical case, $k = 0$, in a larger framework and will help us better understand Newton's original approach.

Thin knotted vortex tubes in stationary solutions to the Euler equation

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Abstract

I will outline some recent results concerning the existence of steady solutions to the Euler equation in $\mathbb{R}^3$ with a prescribed set of (possibly knotted and linked) thin vortex tubes. The role of Beltrami fields will be emphasized throughout.

On construction of time functions

Albert Fathi

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Abstract

This is a joint work with Antonio Siconolfi. The purpose of this lecture is to introduce a different way to obtain Lipschitz time functions on causal Lorentzian manifolds. We will show that they can be obtained from a kind of degenerate auxiliary Finsler metric. We will also explain how this leads to the existence of smooth time functions. A feature of this approach is that it works more generally for fields of cones on a manifold. Our approach evolved from our work on smooth sub-solutions of the Hamilton-Jacobi Equation and the relevance of the Aubry set to this problem. However no prior knowledge of this last subject will be used in the lecture.

Dynamical vs. Thermodynamical (In)stability of Black Objects in Gravity

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Abstract

Understanding the stability properties of black objects is both a very important, but also a very complex problem in general relativity and its higher dimensional generalizations. Based on the well-known dictionary between black objects and thermodynamics, it is natural to come up with criteria for the (in)stability of black objects akin to the standard criteria involving the "specific heat" in the context of phenomenological thermodynamics. A key question is what such notions have to do with notions of instability based on the existence of "growing modes" of the corresponding perturbed Einstein equations.

In this talk, I review the "canonical energy method", which, as I argue, provides a beautiful and clear link between these different concepts of stability (and also a direct connection to the recently proposed approach via a "local Penrose inequality"). I outline some applications of the canonical energy method, such as a proof of the Gubser-Mitra conjecture for black branes, and a connection between the stability of rotating higher dimensional black holes and that of their associated near horizon geometries, thereby proving a recent conjecture of Durkee-Reall.

On the so-called "tunnelling interpretation" of black-hole radiation from the algebraic QFT viewpoint

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Abstract

"Tunnelling processes" through black hole horizons have been investigated in the framework of WKB theory, discovering an interesting interplay with Hawking radiation. We instead [1] adopt the point of view of algebraic QFT in curved spacetime. Using a suitable scaling limit towards a Killing horizon we obtain the leading order of the correlation function relevant for the tunnelling. The computation is done for a certain large class of reference quantum states for scalar fields, including Hadamard states. In the limit of sharp localization on opposite sides of the horizon, the quantum correlation functions appear to have thermal nature with characteristic temperature given by Hawking's one. The approach is valid for stationary charged rotating non-extremal black holes and also covers the case of a Killing horizon which just temporarily exists in some finite region. These results provide strong support to the idea that the Hawking radiation is actually related to phenomena associated to local Killing horizons. Finally, [2] the theory is applied to the model of a ϕ^3 self-interaction in Rindler spacetime, and renormalization counterterms are computed, obtaining that Hawking radiation perturbatively survives the introduction of the interaction.

References

[1] V. Moretti, N. Piamonti: *State independence for tunnelling processes through black hole horizons and Hawking radiation.* Comm. Math. Phys. **309**, 295–311 (2012).

[2] G. Collini, V. Moretti, N. Piamonti: *Tunnelling black-hole radiation with ϕ^3 self-interaction: one-loop computation for Rindler Killing Horizons.* Lett. Math. Phys. **104**, 217–232 (2014).

Boundary dynamics and topology change in Quantum Mechanics

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Abstract

In this talk we will show how one can use the boundary conditions to drive the evolution on a Quantum Mechanical system. We will see how this problem can be expressed in terms of the time dependent Schrödinger equation. In particular we will need the theory of self-adjoint extensions of differential operators in manifolds with boundary. An introduction of the latter as well as meaningful examples will be given. It is known that different boundary conditions can be used to describe different topologies of the associated quantum systems. We will use the previous results to study how this topology change can be accomplished in a dynamical way.

III. Contributed talks

New examples of capillary surfaces in polyhedral regions

Antonio Alarcón[a] and Rabah Souam[b]

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Abstract

Consider a closed region $\mathcal{B}$ in the Euclidean space $\mathbb{R}^3$. A capillary surface in $\mathcal{B}$ is a compact *H-surface* (i.e., with constant mean curvature H) with non-empty boundary, which is $\mathcal{C}^1$ up to the boundary and meeting the frontier $\partial\mathcal{B}$ of $\mathcal{B}$ at a constant angle $\theta \in [0, \pi]$ along its boundary. Capillary surfaces are stationary surfaces for an energy functional under a volume constraint. In the physical interpretation, capillary surfaces model incompressible liquids inside a container in the absence of gravity.

In this talk we will provide a large new family of embedded capillary surfaces inside convex polyhedral regions in the Euclidean space. The angle of contact of the examples we will give is prescribed to be any value in $(\frac{\pi}{2}, \pi]$ and it is allowed to vary from one boundary component to the other (in the physical interpretation, one allows the bounding faces of the polyhedral container to be composed of different homogeneous materials). We will also discuss a classification result for these examples. This talk is based on the papers [1, 2] by the authors.

References

[1] A. Alarcon, R. Souam: *The Minkowski problem, new constant curvature surfaces in* $\mathbb{R}^3$*, and some applications.* J. Reine Angew. Math., in press.

[2] A. Alarcon, R. Souam: *Capillary surfaces inside polyhedral regions.* Preprint, January 2014.

On the existence of geodesics on globally hyperbolic spacetimes with a lightlike Killing vector field

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Abstract

Given a globally hyperbolic spacetime endowed with a complete lightlike Killing vector field and a complete Cauchy hypersurface, we characterize the points which can be connected by geodesics. A straightforward consequence is the geodesic connectedness of globally hyperbolic generalized plane waves with a complete Cauchy hypersurface.

References

[1] R. Bartolo, A.M. Candela, J.L. Flores: *Connection by geodesics on globally hyperbolic spacetimes with a lightlike Killing vector field*. arXiv:1405.0804[math.DG], 2014.

[2] A.M. Candela, J.L. Flores, M. Sánchez: *Global hyperboliticity and Palais–Smale condition for action functionals in stationary spacetimes*. Adv. Math. **218**, 515-536 (2008).

[3] F. Giannoni, P. Piccione: *An intrinsic approach to the geodesical connectedness of stationary Lorentzian manifolds*. Comm. Anal. Geom. **1**, 157-197 (1999).

The classification of naturally reductive homogeneous spaces in dimensions $n \leq 6$

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Abstract

We present a new method for classifying naturally reductive homogeneous spaces – i.e. homogeneous Riemannian manifolds admitting a metric connection with skew torsion that has parallel torsion and curvature. This method is based on a deeper understanding of the holonomy algebra of connections with parallel skew torsion on Riemannian manifolds and the interplay of such connections with the geometric structure on the given Riemannian manifold. We reproduce by much easier arguments the known classifications in dimensions 3, 4 and 5, and obtain as a new result the complete classification in dimension 6. In each dimension, we also exhibit a 'hierarchy' of degeneracy for the torsion form, which we then treat case by case. For the complete degenerate cases, we prove results that are dimension independent. In some situations, we are able to show that any Riemannian manifold with parallel skew torsion has to be naturally reductive. We prove that a 'generic' parallel torsion form defines a quasi-Sasaki struture in dimension 5 and an almost complex structure in dimension 6.

References

[1] I. Agricola, A.C. Ferreira, T. Friedrich: *The classification of naturally reductive homogeneous spaces in dimensions* $n \leq 6$. (To appear).

The Tulczyjew triple in field theories of order higher than one

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Abstract

The geometrical structure known as Tulczyjew triple was used with success in analytical mechanics and first order field theory to describe wide range of systems including systems with constraints or with singular Lagrangians. Starting from basic concepts of variational calculus we derive Tulczyjew triples for the theory of arbitrary given order. The first triple appears as a result of considering a theory of order $k+1$ as constrained first order theory with configurations being jets of order k. Constructing the second triple we use reduction to get rid of some nonphysical degrees of freedom. This picture of higher order field theory is covariant and complete: it contains both Lagrangian and Hamiltonian formalisms as well as Euler-Lagrange equations. Geometry of jet bundles is affine rather than linear therefore we have to use the notion of affine duality and affine phase space. The theory is ilustrated by physical examples.

Trigroups and M-theory

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Abstract

A trigroup is a tricategory with one object and all 1-, 2- and 3-morphisms invertible [1]. M-theory is a mysterious branch of physics believed to unify the five 10D string theories and 11D supergravity. Without assuming any knowledge of physics, we will describe how the mathematics of M-theory leads naturally to a certain trigroup [2].

References

[1] N. Gurski, *An algebraic theory of tricategories.* Ph.D. thesis, Department of Mathematics, University of Chicago, 2006.

[2] J. Huerta, *Division Algebras, Supersymmetry and Higher Gauge Theory.* Ph.D. Thesis, Department of Mathematics, UC Riverside, 2011. Available as arXiv:1106.3385.

CFT, strings and Moonshine

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Abstract

Thirty years ago some very surprising relations between the Fourier coefficients of the Jacobi Hauptmodul or the J-function and representations of the largest finite simple sporadic group, the Monster were discovered. Precise formulation of these relations is now called the Monstrous Moonshine. It has given rise to a large body of new research relating conformal field theory and string theory in physics to geometry and topology in mathematics. We will survey these recent developments and indicate directions for future research.

Geometric Hamiltonian formulation of Quantum Mechanics on complex projective spaces

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Abstract

The topic of the talk is the geometric Hamiltonian formulation of finite-dimensional Quantum Mechanics and its interplay with the standard formulation. In the Quantum-Hamiltonian picture, the phase space is chosen to be the complex projective space $\mathcal{P}(\mathcal{H})$ constructed out of the Hilbert space $\mathcal{H}$ of the considered quantum theory. I focus on the problem of associating quantum states (density matrices on $\mathcal{H}$) to Liouville densities and quantum observables (self-adjoint operators on $\mathcal{H}$) to real scalar functions on phase space $\mathcal{P}(\mathcal{H})$ in order to obtain a classical-like theory. Within an axiomatic approach I construct a general prescription for associating a real scalar function f_A on $\mathcal{P}(\mathcal{H})$ to every quantum observable A such that the dynamics given by the Hamiltonian vector field on $\mathcal{P}(\mathcal{H})$ defined by f_H is equivalent to the dynamics given by the Schrödinger equation given by the Hamiltonian operator H. For this purpose, the Kähler structure of the complex projective space is exploited, the so-called *frame functions* are defined as crucial tools, the Liouville measure on $\mathcal{P}(\mathcal{H})$ (quantum expectation values are computed as integrals w.r.t. this measure in a classical-like fashion) is introduced. I describe the C*-algebra of quantum observables in terms of classical-like observables and I discuss how this framework can be applied to study composite quantum systems and quantum esntanglement. Finally, some ideas to extend the considered formulation to infinite-dimensional case are discussed.

References

[1] V. Moretti, D. Pastorello: *Generalized Complex Spherical Harmonics, Frame Functions, and Gleason Theorem.* Ann. Henri Poincaré **14**,1435-1443 (2013).

[2] V. Moretti, D. Pastorello: *Frame functions in finite-dimensional Quantum Mechanics and its Hamiltonian formulation on complex projective spaces.* Submitted paper, arXiv:1311.1720.

Conformal Killing vectors and virial theorems

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Abstract

A virial theorem is formulated for a Lagrangian mechanical system on a Riemannian manifold and is given in generalised coordinates (see e.g. [1] and references therein and also [2] for a more general approach in Lie algebroid formalism). The special case of an affine virial function is studied and a spherical geometry problem is analysed using this approach. The particular cases of an affine virial function associated to a Killing vector field on the base manifold, homothetic or conform Killing are considered and virial theorems are established for each case.

References

[1] J.F. Cariñena, F. Falceto, M.F. Rañada: *A geometric approach to a generalized virial theorem.* J. Phys. A: Math. Theor. **45**, 395210 (19pp) (2012).

[2] J.F. Cariñena, I. Gheorghiu, E. Martínez, P. Santos: *Virial theorem in quasi-coordinates and Lie algebroid formalism.* Int. J. Geom. Methods Mod. Phys. **11**, 1450055 (9pp) (2014).

Symplectic harmonicity and generalized coeffective cohomologies

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Abstract

Let (M^{2n}, ω) be a symplectic manifold. The notion of *symplectically harmonic form* was introduced by Brylinski as a form α such that $d\alpha = 0 = d * \alpha$, where $*$ is the symplectic star operator. Mathieu [2] proved that every de Rham cohomology class has a symplectically harmonic representative if and only if (M^{2n}, ω) satisfies the Hard Lefschetz Condition. If not, the number of de Rham cohomology classes containing harmonic representative is not determined by the topology of M. In fact, this number can vary if different symplectic structures are considered on M. When this occurs, the manifold is said to be *flexible*. Recently, Tseng and Yau have introduced other cohomologies on symplectic manifolds that admit unique harmonic representative within each class and showed that there exist primitive cohomologies associated with them such that their dimensions can vary with the class of the symplectic form, giving rise to another notion of flexibility, [3]. Additional symplectic invariants of cohomological type were introduced by Bouché [1] using *coeffective forms*, i.e. forms α such that $\alpha \wedge \omega = 0$. In this context it is possible to talk about coeffective flexibility.

In the present talk we relate all these symplectic cohomologies and we give conditions in low dimensions ensuring that all these notions of flexibility are equivalent.

References

[1] T. Bouché: *La cohomologie coeffective d'une variété symplectique.* Bull. Sci. Math. **114(2)**, 115–122 (1990).

[2] O. Mathieu: *Harmonic cohomology classes of symplectic manifolds.* Comment. Math. Helv. **70** 1–9 (1995).

[3] L.-S. Tseng, S.-T. Yau: *Cohomology and Hodge theory on symplectic manifolds: I and II.* J. Differential Geom. **91**, 383–416, 417–444 (2012).

IV. Posters

Hypersymplectic and hyperkähler structures with torsion on Lie algebroids

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Abstract

We define hypersymplectic structures with torsion (HST) on Lie algebroids and establish a 1-1 correspondence between hyperkähler structures with torsion (HKT) and HST structures. We also obtain a contravariant definition of both HKT and HST structures on Lie algebroids.

References

[1] P. Antunes, J. M. Nunes da Costa, *Hyperstructures on Lie algebroids*, Rev. in Math. Phys. 25 (2013), no. 10, 1343003 (19 pages).

[2] P. Antunes, J. M. Nunes da Costa, *Hyperstructures with torsion on Lie algebroids*, in preparation.

[3] M. Cabrera, A. Swann, *The intrinsic torsion of almost quterninon-hermitian manifolds*, Ann. Inst. Fourier, Grenoble 58 (2008), 1455–1497.

[4] P.S. Howe, G. Papadopoulos, *Twistor spaces for hyper-Kähler manifolds with torsion*, Phys. Lett., B 379 (1996), 80–86.

Hypersymplectic and hyperkähler structures with torsion: from Lie to Courant algebroids

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Abstract

We show that hypersymplectic structures on Courant algebroids encompass hyperkähler and hyperkähler structures with torsion on Lie algebroids. Cases of hypersymplectic structures on Courant algebroids which are doubles of Lie, quasi-Lie and proto-Lie bialgebroids are investigated.

References

[1] P. Antunes, J. M. Nunes da Costa, *Hyperstructures on Lie algebroids*, Rev. in Math. Phys. 25 (2013), no. 10, 1343003 (19 pages).

[2] H. Bursztyn, G. Cavalcanti and M. Gualtieri, *Generalized Kähler and hyper-Kähler quotients*, Poisson geometry in mathematics and physics, Contemp. Math., **450**, 61–77, Amer. Math. Soc., Providence, RI, (2008).

[3] P.S. Howe, G. Papadopoulos, *Twistor spaces for hyper-Kähler manifolds with torsion*, Phys. Lett., B 379 (1996), 80–86.

Lightlike relaxed elastic lines in Minkowski space

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Abstract

The optimality for a lot of discipline is an important. The search for extremes is the subject of research for mountaineers, scientists, mathematicians, physicists, biologists. So, the calculus of variation was developed with effects of the search. One of these effects is relaxed elastic line. Because, a curve which minimizes the value of energy density is called relaxed elastic line. In this study, we analyzed relaxed elastic line in Minkowski space which is the mathematical modeling of Einstein's relativity theory. For this space, when surface is lightlike and the curve is lightlike, we calculated necessary boundary conditions to be extremal of the curve. So, we examined all of situations and we obtained the results by taking advantage from the relationship between the curve and the surface. Then, we found some differential equations which determine the curve. It is the difference from the others since its velocity vector is lightlike. Thus, this study can lead to physicists.

References

[1] K. L. Duggal, A. Bejancu: *Lightlike submanifolds of semi-Riemannian manifolds and applications*. Kluwer Academic Publishers: 1996.

[2] G. S. Manning: *Relaxed elastic line on a curved surface*. Quarterly of Applied Mathematics. **45**, 515 (1987).

[3] N. Gürbüz: *A variation problem for null curves*. Adv. Theor. Appl. Mech. **3**, 67–73 (2010).

An exactly solvable deformation of the Coulomb problem from a quantum Taub–NUT system

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Abstract

In this contribution we show that the quantization of a certain maximally superintegrable Hamiltonian $\mathcal{H}_\eta$ [1] defined on an N-dimensional Taub–NUT space [2] leads to a new exactly solvable deformation of the quantum Coulomb problem [3].

The quantization of $\mathcal{H}_\eta$ will be performed in such a way that the maximal superintegrability of the classical Taub–NUT Hamiltonian will be fully preserved. In particular, we will prove that this strong condition is fulfilled by the so-called conformal Laplace–Beltrami quantization prescription, where the conformal Laplacian contains the usual Laplace–Beltrami operator on the Taub–NUT manifold plus a term proportional to its scalar curvature.

In this way, the eigenvalue problem for the quantum counterpart of $\mathcal{H}_\eta$ can be rigorously solved, and it is found that its discrete spectrum is just a smooth deformation (in terms of the Taub–NUT parameter η) of the ND Coulomb spectrum. Moreover, it turns out that the maximal degeneracy of the Coulomb system is preserved under deformation.

References

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Geodesics in static cylindrically symmetric vacuum spacetimes with a cosmological constant

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Abstract

We consider static cylindrically symmetric vacuum spacetimes, the Levi-Civita spacetime and the Linet-Tian spacetime [1][2], which includes a cosmological constant. We analyse the geodesics of the Linet-Tian spacetime and compare their dynamics with those of the geodesics in the Levi-Civita spacetime. In particular, we study the effects that the introduction of a cosmological constant has on the orbits' stability [3].

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Dual rigidity results for the sphere and the hyperbolic plane

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Abstract

By means of a purely synthetic technique, we get a new rigidity result for the sphere without any curvature conditions, nor completeness or compactness. As well as a dual result for the hyperbolic plane, the spacelike sphere in the Minkowski space.

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Geometry and topology of cosymplectic spheres

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Abstract

The notion of cosymplectic structure was introduced by P. Libermann in the late 50s as a pair (η, Ω), where η is a closed 1-form and Ω a closed 2-form on an $2n+1$-dimensional manifold M, such that $\eta \wedge \Omega^n$ is a volume form. Cosymplectic manifolds play an important role in the geometric description of time-dependent mechanics (see [1] and references therein). Starting from 1967, when Blair defined an adapted Riemannian structure on a cosymplectic manifold, a study of the metric properties on these manifolds was also initiated.

We study the geometry and topology of cosymplectic circles and cosymplectic spheres, which are the analogues in the cosymplectic setting of contact circles and contact spheres, introduced by Geiges and Gonzalo ([2]), and then generalized by Zessin ([2]). We provide a complete classification of 3-dimensional compact manifolds that admit a cosymplectic circle.

We introduce the notion of tautness and of roundness for a cosymplectic p-sphere. To any taut cosymplectic circle on a three-dimensional manifold M we are able to associate canonically a complex structure and a conformal symplectic couple on $M \times \mathbb{R}$.

In dimension three a cosymplectic circle is proved to be round if and only if it is taut. In higher dimensions we provide examples of cosymplectic circles which are taut but not round and examples of cosymplectic circles which are round but not taut. Finally we show that the three cosymplectic structures of any 3-cosymplectic manifold generate a cosymplectic sphere which is both round and taut.

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The modular class of a Dirac map

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Abstract

Dirac manifolds are Lie algebroids and they have an associated modular class which lives in the first cohomology group of its Lie algebroid cohomology [2]. Two types of morphisms between Dirac manifolds are known: forward Dirac maps and backwards Dirac maps. In this work we introduce the modular class of a Dirac map between regular Dirac manifolds which generalizes the modular class of a Poisson map [1] and makes use of the modular class of a skew-algebroid relations introduced in [3].

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Lightlike sets and applications

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Abstract

Extending a structure introduced by G. Galloway in [1], we define the notion of *lightlike sets* in spacetimes, which comprise, under a single label, such diverse objects as null geodesics in strongly causal spaces, (locally) achronal smooth null submanifolds, geodesic rays and (portions of) causal boundaries. In the first part of this presentation, we prove a number of general structural results about lightlike sets which have independent interest. In the second part, we apply some of these results to establish the following theorem, related to a conjecture by Ehlers and Kundt in [2]. Assume that $(\mathbb{R}^4, g)$ is a Ricci-flat globally hyperbolic, future null geodesically complete *wave-type* spacetime, with metric

$$g(.,.) = g_0(.,.) + 2dudv + H(x,u)du^2,$$

where g_0 is the standard flat metric on $\mathbb{R}^2$, the variables (v,u) are the standard coordinates of $\mathbb{R}^2$, and $H : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}$ is a smooth real function. Given $(v,u) \in \mathbb{R}^2$, and let $\Sigma_{v,u} = \{(x,v,u) \in M \mid x \in \mathbb{R}^2\}$. Suppose, in addition, that there exists a future-directed null $\Sigma_{u,v}$-ray for each u, v in some neighbourhood. Then (M,g) is isometric to Minkowski spacetime.

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Riemannian connections with torsion adapted to almost CR structures

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Abstract

We describe a class of Riemannian almost CR manifolds which constitute a natural generalization of Kähler, Sasakian and cosymplectic manifolds [2].

A Riemannian almost CR manifold is an almost CR manifold (M, HM, J) endowed with a compatible Riemannian metric g. We provide necessary and sufficient conditions for such a manifold to admit a *characteristic* connection, i.e. a metric connection with totally skew-symmetric torsion [1] parallelizing the almost CR structure. The obtained result generalizes known results of Friedrich and Ivanov, concerning the classes of almost Hermitian and almost contact metric manifolds [3]. Examples of Riemannian almost CR manifolds with torsion having CR codimension higher than 1 come from the theory of homogeneous naturally reductive spaces, standard CR manifolds, f-structures, 3-Sasakian manifolds, complex contact manifolds, and from the geometry of CR submanifolds.

We focus on a natural construction involving principal fiber bundles, leading to new classes of examples. We also discuss the influence of the curvature of the metric on the underlying almost CR structure. Special attention is reserved to the case where the manifold admits a flat characteristic connection.

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Flat almost complex surfaces in $S^3 \times S^3$

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Abstract

This work discusses almost complex surfaces in the nearly Kähler manifold $S^3 \times S^3$. We briefly recall that a nearly Kähler manifold is a manifold with a Riemannian metric g and a compatible almost complex structure J such that the tensor field ∇J is skew-symmetric. The manifold $S^3 \times S^3$ admits such a nearly Kähler structure; the nearly Kähler metric on $S^3 \times S^3$ is not the usual product metric. Almost complex surfaces in a nearly Kähler manifold are surfaces whose tangent spaces are invariant under the almost complex structure, that is, J maps tangent vectors onto tangent vectors.

In a previous paper [1] the following correspondence theorem was proven: locally an almost complex surface in the nearly Kähler manifold $S^3 \times S^3$ corresponds to a surface ϵ in $\mathbb{R}^3$ that satisfies the Wente H-equation

$$\epsilon_{uu} + \epsilon_{vv} = -\frac{4}{\sqrt{3}}\epsilon_u \times \epsilon_v,$$

and vice versa. Also, on each almost complex surface in $S^3 \times S^3$ there is a holomorphic quadratic differential $\Lambda\, dz^2$ corresponding to the differential $\langle \epsilon_z, \epsilon_z \rangle\, dz^2$ on ϵ.

In [2] we have proven a Bonnet-type existence and uniqueness theorem for almost complex surfaces in $S^3 \times S^3$. The proof uses the previously mentioned correspondence theorem. Secondly a classification of all the flat almost complex surfaces is given: there is a 2-parameter family of these flat surfaces and explicit parametrizations can be obtained.

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On complete spacelike hypersurfaces in Lorentzian product spaces

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Abstract

We deal with two-sided complete hypersurfaces immersed in a Riemannian product space, whose base is supposed to have sectional curvature bounded from below. In this setting, we obtain sufficient conditions which assure that such a hypersurface is a slice of the ambient space, provided that its angle function has some suitable behavior. Furthermore, we establish a natural relation between our results and the classical problem of to describe the geometry of a hypersurface immersed in the Euclidean space through the behavior of its Gauss map.

Keywords: Mean Curvature, Riemannian Products Spaces, CMC Hypersurfaces

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Extendibility of geodesics on transverse Riemann-Lorentz manifolds with polar end

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Abstract

We study the extendibility of geodesics on a transverse Riemann-Lorentz typechanging manifold with polar end, a manifold endowed with a certain metric that fails to be defined at the hypersurface of signature change. We prove the existence and uniqueness of pregeodesics going across the hypersurface at each point in a single direction, called polar normal direction.

Key words: Riemann-Lorentz Geometry, geodesic

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Lorentzian homogeneous gradient Ricci solitons: their Ricci tensor

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Abstract

Let (M, g) be a Lorentzian manifold of dimension $n + 2$ for $n \geq 1$, let ρ be the *Ricci tensor*.

The triple (M, g, f) is said to be a *Lorentzian gradient Ricci soliton* if $f \in C^\infty(M)$ satisfies the *gradient Ricci soliton equation*:

$$\mathrm{Hess}_f + \rho = \lambda\, g \text{ for some } \lambda \in \mathbb{R}\,, \tag{1}$$

where Hess_f is the Hessian and f is often called the *potential function*. Setting $f = 0$ yields the *Einstein equation* $\rho = \lambda g$; thus Equation (1) is a natural generalization of the Einstein equation and a gradient Ricci soliton can be thought of as a generalized Einstein manifold. Gradient Ricci solitons also correspond to self-similar solutions of the *Ricci flow* $\partial_t g(t) = -2\rho_{g(t)}$. For these reasons, gradient Ricci solitons have been extensively investigated in the literature. If $\lambda > 0$ (resp. $\lambda = 0$ or $\lambda < 0$), then (M, g, f) is said to be *shrinking* (resp. *steady* or *expanding*).

We describe the structure of the Ricci tensor on a locally homogeneous Lorentzian gradient Ricci soliton. In the non-steady case, we show the soliton is rigid in dimensions three and four. In the steady case, we give a complete classification in dimension three.

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Hidden symmetries of geometrical structures of the classical phase space of general relativistic test particle

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Abstract

A *classical spacetime* is assumed to be an oriented and time oriented 4–dimensional manifold equipped with a Lorentzian metric.

In literature as symmetries (eventually *hidden symmetries*) of the classical phase space are usually considered infinitesimal symmetries of the kinetic energy function on the cotangent bundle of the spacetime. It is very well known, [3], that such symmetries are given as the Hamiltonian lifts (with respect to the canonical symplectic 2–form) of functions constant of motions. Functions constant of motions which are polynomial on fibres of the cotangent bundle are given by Killing k–vector fields, $k \geq 1$. For $k = 1$ the corresponding infinitesimal symmetries are the flow lifts of Killing vector fields and so they are projectable on infinitesimal symmetries of the spacetime. For $k \geq 2$ the corresponding infinitesimal symmetries are not projectable and they are called *hidden symmetries*.

On the other hand the phase space of general relativistic test particle can be defined either as the observer space (a part of the unit pseudosphere bundle given by time-like future oriented vectors) or as the 1-jet space of motions, [2]. The metric and an electromagnetic fields then define a geometrical structures given by a 1–form and a closed 2–form. As phase infinitesimal symmetries we define infinitesimal symmetries of the geometrical structure of the phase space. Phase infinitesimal symmetries which are projectable on the spacetime were studied on the observer space by Iwai [1] and on 1–jet space of motions by Janyška and Vitolo [2]. In both situations projectable symmetries are given by the flow lifts of Killing vector fields. In the lecture we describe hidden (nonprojectable) infinitesimal symmetries for the phase space given as the 1-jet space of motions. We prove that hidden symmetries are given by the Hamilton–Jacobi lifts of conserved phase functions and we give explicit construction of hidden symmetries generated by Killing (eventually Killing–Maxwell) multi–vector fields.

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A new construction of Lagrangians in the complex Euclidean plane

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Abstract

An isometric immersion $\phi : M^n \to \widetilde{M}^n$ of an n-dimensional Riemannian manifold M^n into an n-dimensional Kaehler manifold $\widetilde{M}^n$ is said to be Lagrangian if the complex structure J of $\widetilde{M}^n$ interchanges each tangent space of M^n with its corresponding normal space. Lagrangian submanifolds appear naturally in several contexts of Mathematical Physics.

The aim of the poster is to present a recent contribution (see [1]) to the study of a new method to construct a large family of Lagrangian surfaces in complex Euclidean plane $\mathbb{C}^2$ in terms of two planar curves $\alpha = \alpha(t) \subset \mathbb{C} \setminus \{0\}$ and $\omega = \omega(s) \subset \mathbb{C} \setminus \{0\}$

$$\phi = \alpha * \omega : I_1 \times I_2 \subset \mathbb{R}^2 \to \mathbb{C}^2 = \mathbb{C} \times \mathbb{C}$$
$$\phi(t,s) = \left(\int_{s_0}^{s} \dot{\omega}(y)\overline{\omega(y)}dy - \int_{t_0}^{t} \alpha'(x)\overline{\alpha(x)}dx, \alpha(t)\,\omega(s) \right),$$

where $'$ and $\dot{}$ denote the derivatives respect to t and s respectively.

Among the Lagrangians in this family, we characterize those which are i) minimal, ii) of constant mean curvature, iii) Hamiltonian stationary, iv) solitons for mean curvature flow, and v) Willmore. These characterizations are given in terms of simple properties of the curvatures of the generating curves. As an application, we provide explicitly conformal parametrizations of known and new examples of these classes of Lagrangians in $\mathbb{C}^2$.

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Fock quantization of field theories with boundary degrees of freedom

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Abstract

We use the geometric constraint algorithms developed by Gotay, Nester and Hinds to study in a rigorous way the Hamiltonian formulation of a simple $1 + 1$ dimensional field theory coupled to point-particle degrees of freedom. An important result of this analysis is the identification of the generalized submanifold in an appropriate cotangent bundle where the Hamiltonian dynamics takes place. By using this as the space of classical solutions to the equations of motion of the system we quantize the system by constructing the Fock space and discuss relevant operators associated with boundaries and their use to understand the quantum dynamics of the system.

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New examples of paracontact metric manifolds without contact metric counterpart

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Abstract

Paracontact metric (κ, μ)-spaces (M, ϕ, ξ, η, g) satisfy that $h^2 = (\kappa - 1)\phi^2$. Therefore, $\kappa \leq 1$ and $\kappa = 1$ if and only if the manifold M is Sasakian, [1].

On the other hand, paracontact metric (κ, μ)-spaces (M, ϕ, ξ, η, g) satisfy $h^2 = (\kappa + 1)\phi^2$, but this does not impose any restrictions on the possible values of κ because the metric g is now pseudo-Riemannian. However, it is useful to distinguish three cases: $\kappa > -1$, $\kappa < -1$ and $\kappa = -1$. The first two have been the object of study by several authors (see [2], for instance) but only some examples exist of the last one, which has no contact metric counterpart.

We will provide a local classification of the paracontact metric $(-1, \mu)$-spaces in terms of the rank of h and we will present examples with every possible constant rank (see [3]).

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Non-commutative Quantum Gravity

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Abstract

Motivated by topics in quantisation of the non-commutative standard model and the introduction of algebraic techniques and concepts into quantum gravity (following for example Crane, Baez [1] and Barrett), we define spectral C*-categories [2], which are deformed spectral triples in a sense we will make precise. This definition gives representations of a C*-(in particular a category of Morita equivalence bimodules) on a small category of Hilbert spaces and linear maps, the interpretation of a topological quantum field theory, TQFT. The construction passes two mandatory tests: (i) we prove a classical limit theorem producing a Riemannian spin manifold manifesting Connes' non-commutative counterpart of Einstein's equivalence principle and (ii) we confirm with a litmus test, a consistency with the fermion mass matrix. If time permits, we will present an algebra invariant taking the form of a partition function arising from a C*-bundle dynamical system and then address other similar topics connecting spectral triples with quantum gravity.

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(2+1) Non-commutative spacetimes with a cosmological constant

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Abstract

First, the Drinfel'd double (DD) structure underlying a one-parameter family AdS_ω of (2+1) relativistic Lie algebras is presented (AdS corresponds to $\omega = 1$, dS to $\omega = -1$ and Poincaré to $\omega = 0$, ω being related to the cosmological constant). Next, by using the fact that the 'exotic' (2+1) Galilean and Newton-Hooke Lie algebras (with either zero or non-zero ω, respectively) originate as a well-defined non-relativistic contraction of a two-fold centrally extended AdS_ω algebra (Poincaré, for the former; dS, for the latter), we explicitly unveil the DD structure underlying both (2+1) Galilean and Newton-Hooke algebras.
In both scenarios, relativistic and non-relativistic, the underlying non-commutative spacetimes, along with the corresponding full quantum group structure, are analysed.
Finally, a tentative generalisation of this programme to (3+1) dimensions is briefly outlined. The motivation behind this analysis stems from the highly suspected role that non-commutative spacetimes ought to play in a quantum theory of gravity.

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Curvature identities and Gauss-Bonnet type theorems

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Abstract

For a fixed $n \in \mathbb{N}$, the curvature tensor of a pseudo-Riemannian metric, as well as its covariant derivatives, satisfy certain identities that hold on any manifold of dimension less or equal than n. As an example, on any pseudo-Riemannian manifold of dimension 2 the following relation holds:

$$Ric - \frac{r\,g}{2} = 0 \,, \tag{2}$$

where Ric denotes the Ricci tensor and r the scalar curvature of g.

In this poster, we re-elaborate recent results by Gilkey-Park-Sekigawa regarding these curvature identities on pseudo-Riemannian manifolds (see [3]). To this end, we use the classical theory of natural operations, that allows us to simplify some arguments and to generalize some results of [1], both by dropping a symmetry hypothesis and by including p-covariant curvature identities, for any even p.

In the final section, we state how to use this theorem to refine a classical result by Gilkey ([1]), regarding the uniqueness of the Gauss-Bonnet theorem.

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Finite element methods and its applications

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Abstract

The finite element method (FEM) is a numerical technique for finding approximate solutions to boundary value problems for differential equations by using variational methods (the calculus of variations) to minimize an error function and produce a stable solution in mathematical point of view. FEM was founded by A. Hrennikoff and R. Courant to solve complex elasticity and structural analysis problems in civil and aeronautical engineering. The method has since been generalized for the numerical modeling of physical systems in a wide variety of engineering disciplines, e.g., electromagnetism, heat transfer, and fluid dynamics.

This study will explain the basics of FEM and its applications.

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Results on real hypersurfaces in non-flat complex space forms in terms of the *-Ricci tensor

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Abstract

The *-**Ricci tensor**, S^*, of real hypersurfaces in non-flat complex space forms was studied by Hamada in [1]. The aim of the present poster is to provide results on three dimensional real hypersurfaces in non-flat complex space forms, when the *-Ricci tensor of them satisfies conditions of parallelism [2]. Furthermore, the notion of ***-Ricci soliton** will be given and results concerning real hypersurfaces admitting this new type of soltion will be included [3].

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Variational principles for multisymplectic second-order classical field theories

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Abstract

We state a unified geometrical version of the variational principles which enables us to derive the Lagrangian and Hamiltonian equations for these kinds of systems. Then, the standard Lagrangian and Hamiltonian formulations of these principles and the corresponding field equations are recovered from this unified framework.

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New properties of the Nambu bracket. Applications

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Abstract

In the seventies Nambu in [1] proposed a new approach to the classical dynamics based on an N dimensional NambuPoisson manifold replacing the even dimensional Poisson manifold and on $N-1$ Hamiltonian $H_1, \ldots, H_{N-1}$ instead of a single Hamiltonian H. In Nambu's formulation, the Poisson bracket is replaced by the Nambu bracket. Although the Nambu formalism is a generalization of the Hamiltonian formalism its real applications are not as rich as the applications of this last one.

The objective of this section is to provide new properties of the Nambu bracket. These new properties play an important role in to construction the inverse approach in ordinary differential equations (see [2]) We prove that the Nambu bracket satisfies the identities:

$$\Omega(f_1, \ldots, f_{N-1}, g_1, \ldots, g_N, G) := -\{f_1, \ldots, f_{N-1}, G\}\{g_1, \ldots, g_N\}$$
$$+\left(\sum_{n=1}^{N}\{f_1, \ldots, f_{N-1}, g_n\}\{g_1, \ldots, g_{n-1}, G, g_{n+1}, \ldots, g_N\}\right) = 0,$$
$$F_\lambda(f_1, \ldots, f_{N-1}, g_1, \ldots, g_N) = \sum_{j=1}^{N} \frac{\partial}{\partial x_j}(\lambda\Omega(f_1, \ldots, f_{N-1}, g_1, \ldots, g_N, x_j)) = 0,$$

for arbitrary functions $f_1, \ldots, f_{N-1}, G, g_1, \ldots, g_N, \lambda$. Note that the second identity is a generalization of the Filippov Identity which is obtained when $\lambda = 1$.

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Intelligent spin states constructed from $SU_q(2)$ coherent states

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Abstract

It is shown that $SU_q(2)$ Coherent Spin States behave as Intelligent Spin States on two orthogonal components which are perpendicular to the direction of the mean value of the spin operator.

The symplectic normal space of a Lagrangian fibration

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Abstract

A fibration $\pi : L \to Q$ is Lagrangian if the total space (L, ω) is a symplectic manifold and the fibers are Lagrangian. Assume that a Lie group G acts on L preserving the symplectic and fibered structure and it has an associated momentum map $J : L \to \mathfrak{g}^*$, which is (in general) non-equivariant. In this setting, we will give a description of the symplectic normal space at any point $z \in L$. This result is a generalization of previous results on magnetic cotangent bundles and is a first step towards the construction of semi-local normal forms for this structure.

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Zero mean curvature hypersurfaces in warped product Lorentzian manifolds

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Abstract

We are interested in obtain rigidity results for spacelike hypersurface whose mean curvature vanishes identically (maximal hypersurfaces). The ambient spacetimes has the structure of a warped product of a definite negative 1-dimensional base and a parabolic Riemannian manifold as fiber. Following [2], this class of spacetimes are introduced as spatially parabolic covered GRW spacetimes. Under extrinsic natural assumptions, we get to characterize maximal hypersurfaces as level hypersurfaces of the universal time function. As no direct application, several new Calabi-Bernstein type results are obtained.

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The generalized Landau-Raychaudhuri equation and its applications

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Abstract

Consider an n-dimensional ($n \geq 4$) Lorentzian manifold (M, g) with the metric g having signature $(-, +, \dots, +)$ and the Levi-Civita connection ∇ with its Ricci tensor Ric.

Let ξ be a smooth time-like vector field in a domain U of (M, g). In this case for each point $x \in U$ the tangent vector space T_xM has the orthogonal decomposition $T_xM = H \oplus V$ where $V = \mathrm{span}\,\{\xi_x\}$ and $H = \xi_x^{\perp}$. We define the following tensors $A = \nabla\xi$, $\omega(\,\cdot\,,\cdot\,) = 2^{-1}\,(g(h\,\cdot, Ah\,\cdot) - g(Ah\,\cdot\,, h\,\cdot))$ and $\sigma(\,\cdot\,,\cdot\,) = (n-1)^{-1}\,\theta g(h\,\cdot, h\,\cdot) - 2^{-1}\,(g(h\,\cdot, Ah\,\cdot) + g(Ah\,\cdot\,, h\,\cdot))$ for the orthogonal projections v_x: $T_xM \to V$ and h_x: $T_xM \to H$ and the scalar function $\theta = traceA$. Then the following formula $\operatorname{div} A\xi = Ric(\xi, \xi) - g(\omega, \omega) + g(\sigma, \sigma) + (n-1)^{-1}\theta^2 + \xi(\theta)$ holds (see [1]). This formula is well known as the kinematic Landau-Raychaudhuri equation (see [2, p. 70]) in the case where $n = 4$.

On the other hand, we will call a time-like vector field ξ determined in U of (M, g) as *almost conformal Killing* if the one-parameter diffeomorphism group generated by this vector field consists of local conformal transformations of $\xi^{\perp}$, which means $(\mathcal{L}g)(X, Y) = \alpha g(X, Y)$ for any smooth vector fields X, Y which belong to $\xi^{\perp}$ and a scalar function α.

Based on the generalized Landau-Raychaudhuri equation we can prove some propositions which characterize almost conformal Killing vector fields. In particular, the following proposition holds.

Theorem. An n-dimensional Lorentzian manifold (M, g) with a time-like almost conformal Killing vector field ξ which satisfies the conditions: **1.** there exists a domain U in (M, g) the with the space-like boundary ∂U in which the vector field is everywhere nonzero and is orthogonal to the boundary at all boundary points, **2.** $Ric(\xi, \xi) \leq 0$ in domain U, and **3.** the mean curvature of the boundary is nonnegative and is strictly positive at least at one point doest not exists.

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The positive mass theorem for low-regularity metrics

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Abstract

We show that the positive mass theorem, which was proven for smooth metrics by Schoen and Yau [2] resp. Witten [3] holds for asymptotically flat, n-dimensional Riemannian manifolds with a metric that is continuous, lies in the Sobolev space $W^{2,n/2}_{loc}$, and has non-negative scalar curvature in the distributional sense. Our approach requires an analysis of smooth approximations to the metric, and a careful control of elliptic estimates for a related conformal transformation problem. We describe how an approach similar to the one in [1] leads to the result.

If the metric lies in $W^{2,p}_{loc}$ for $p > n/2$, then we show that our metrics may be approximated locally uniformly by smooth metrics with non-negative scalar curvature.

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Real hypersurfaces with two principal curvatures in complex projective and hyperbolic planes

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Abstract

The following problem was posed by Niebergall and Ryan [1] in the 90's:

> "*Are there hypersurfaces in* $\mathbb{C}P^2$ *or* $\mathbb{CH}^2$ *that have* ≤ 2 *principal curvatures, other than the standard examples?*"

In this talk I show that non-standard examples exist and present the classification of real hypersurfaces with two nonconstant principal curvatures in complex projective and hyperbolic planes [2]. It turns out that each such hypersurface is foliated by equidistant Lagrangian flat surfaces with parallel mean curvature or, equivalently, by principal orbits of a cohomogeneity two polar action.

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A new setting of application of k-symplectic geometry: Lie systems

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Abstract

In this talk we present a new interesting setting of application of k–symplectic geometry: systems of first-order ordinary differential equations, in particular Lie systems.

A *Lie system* is a system of first–order ordinary differential equations describing the integral curves of a t-dependent vector field taking values in a finite-dimensional real Lie algebra of vector fields: a so-called *Vessiot–Guldberg Lie algebra*. We here suggest the definition of a particular class of Lie systems, the *k–symplectic Lie systems*, admitting a Vessiot–Guldberg Lie algebra of Hamiltonian vector fields with respect to the presymplectic forms of a k–symplectic structure. We devise new k-symplectic geometric methods to study their superposition rules, time independent constants of motion and general properties. Our results are illustrated by examples of physical and mathematical interest.

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