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Group Actions on Gauge Theory

Manifolds have been one of central topics of mathematics from the past century. The geometry of manifolds of dimension ≤ 2 has been well understood. The geometry of 3-manifolds has been steadily progressed. The manifold topology of dimension ≥ 5 was developed in the 1960s by the s-cobordism and surgery theories.

There was very little known about 4-manifold geometry through the 1970s. In 1981 M. Freedman classifiled the closed, simply-connected topological 4-mainifolds by the Whitney trick. In 1982 S. Donaldson showed that some topological 4-manifolds have some obstructions to have smooth structures by using gauge theory. The s-cobordism and surgery predictions for smooth 4-manifolds did not hold, resulted in a clash between the theories of smooth and topological manifolds in 4-dimension. The 4-dimension is the only dimension in which a fixed homeomorphism type of closed manifold is represented by infinitely many diffeomorphism types, also there are manifolds homeomorphic but not diffeomoephic to R4. Donaldsons theory of studying the self-dual Yang-Mills equations was central to smooth 4-manifold theory for 12 years, until it was superseded in 1994 by studying the Seiberg-Witten equations, which simplifies and expands Donaldsons theory and results.

The gauge theory came to mathematics from the theoretical physics. The results of gauge theory, from Donaldson through the Seiberg-Witten theories, proves the nonexistence of smooth manifolds satisfying certain constraints, the nonexistence of connected-sum splitting, the nonexistence of diffeomorphisms between pairs of manifolds which are topologically the same, the nonexistence of symplectic sturucture on some manifolds, the Thom conjecture on symplectic 4-manifolds, and so on.

Let a fininte group act on a bundle over a closed, smooth, oriented 4-manifold with a 2-dimensional fixed point submanifold on the base manifold. We will discuss on finite group actions on the moduli space of self-dual connections, the relation between the Donaldson invariant in the equivariant set up and the Donaldson invariant in the quotient set up when the action is an involution, and finite group actions on Seiberg-Witten moduli space.